Day 232 **A Tier 10-a-day: Model Solutions**

1 Bounds in measurement

A circle has a radius of 13 cm, correct to the nearest cm. (Calculator)
Work out the lower bound for the area of the circle.
Give your answer to 3 significant figures.

Lower bound of radius = 12.5 cm

Area = $\pi \times 12.5^2 = 491$

Answer: 491 cm^2

2 Prime factors

Express 2475 as a product of prime factors. Give your answer in index form.

Divide by prime numbers in turn: 2475 ÷ 3 = 825, 825 ÷ 3 = 275, 275 ÷ 5 = 55, 55 ÷ 5 = 11, 11 ÷ 11 = 1
2475 = 3 × 3 × 5 × 5 × 11

In index form: $2475 = 3^2 \times 5^2 \times 11$

Answer: $2475 = 3^2 \times 5^2 \times 11$

3 Forming quadratic equations

A rectangle has width x cm and length (x + 4) cm.
The area of the rectangle is 32 cm^2.
(a) Show that $x^2 + 4x - 32 = 0$
(b) Work out the value of x.

(a) Area = length × width, so $x(x + 4) = 32$

$x^2 + 4x = 32$

$x^2 + 4x - 32 = 0$

(b) Factorise: $(x - 4)(x + 8) = 0$

$x = 4$ or $x = -8$

A length cannot be negative, so $x = 4$

Answer: (a) Shown (b) x = 4

4 Reverse percentages

In a sale, all prices are reduced by 5%.
A tent costs £285 in the sale.
Work out the original price of the tent.

95% represents £285
1% = £285 ÷ 95 = £3
100% = £3 × 100 = £300.00

Answer: £300.00

5 Difference of two squares

(a) Factorise $4y^2 - 25z^2$
(b) Use the same idea to work out $108^2 - 92^2$ without a calculator.

(a) $4y^2 - 25z^2 = (2y)^2 - (5z)^2$

$(2y - 5z)(2y + 5z)$

(b) $a^2 - b^2 = (a + b)(a - b)$, so $108^2 - 92^2 = (108 + 92)(108 - 92)$

$= 200 \times 16 = 3200$

Answer: (a) $(2y - 5z)(2y + 5z)$ (b) 3200

6 Circle theorems: angle at the centre

A, B and C are points on a circle with centre O.
Angle AOB = 150°.
Work out the size of angle x.
Give a reason for your answer.

The angle at the centre is twice the angle at the circumference

$x = 150 \div 2 = 75$

Answer: x = 75°

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7 Profit and loss

Kiran buys a sofa for £200 and sells it making a profit of 15%.
Work out the selling price.

Selling price = 115% of the cost price
£200 × 1.15 = £230
Answer: £230

8 Probability with three selections

A bag contains 7 black pens and 3 blue pens.
Three pens are taken at random without replacement.
Work out the probability that at least one of the three pens is blue.

P(none is blue) = P(all black) = $\frac{7}{10} \times \frac{6}{9} \times \frac{5}{8} = \frac{7}{24}$

P(at least one blue) = $1 - \frac{7}{24} = \frac{17}{24}$

Answer: $\frac{17}{24}$

9 Vector geometry

OAB is a triangle. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.
P is on OA with OP : PA = 2 : 1.
Q is on OB with OQ : QB = 2 : 1.
(a) Find $\overrightarrow{PQ}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) What do you notice about PQ and AB?

(a) $\overrightarrow{OP} = \frac{2}{3}\mathbf{a}$ and $\overrightarrow{OQ} = \frac{2}{3}\mathbf{b}$

$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ} = -\frac{2}{3}\mathbf{a} + \frac{2}{3}\mathbf{b} = \frac{2}{3}(\mathbf{b} - \mathbf{a})$

(b) $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$, so $\overrightarrow{PQ} = \frac{2}{3}\overrightarrow{AB}$

Answer: (a) $\frac{2}{3}\mathbf{b} - \frac{2}{3}\mathbf{a}$ **(b) PQ is parallel to AB and** $\frac{2}{3}$ **times as long**

10 Bounds and accuracy

A cyclist travels 790 metres in 5.2 seconds. The distance is correct to the nearest 10 metres and the time to the nearest 0.1 second.
(a) Work out the upper bound and the lower bound of the speed in m/s.
(b) Write the speed to a suitable degree of accuracy. Give a reason for your answer.

(a) Upper bound = 795 ÷ 5.15 = 154.3689
Lower bound = 785 ÷ 5.25 = 149.5238
(b) Both bounds round to 150 to 2 s.f. but not to the same value to 3 s.f.
So the speed is 150 m/s to 2 significant figures.
Answer: (a) 154.3689, 149.5238 (b) 150 m/s (2 s.f.)