Day 227 **A Tier 10-a-day: Model Solutions**

1 Fractions of an amount

Oscar spends $\frac{2}{5}$ of the money on a book and $\frac{1}{3}$ of the money that is left on a game.

What fraction of the money does Oscar have left?
Give your answer in its simplest form.

After the first purchase $\frac{3}{5}$ of the money is left.

$\frac{1}{3}$ of that is spent, so $\frac{2}{3}$ of it is kept.

$\frac{3}{5} \times \frac{2}{3} = \frac{2}{5}$

Answer: $\frac{2}{5}$

2 Equations with brackets

Solve $6(x - 3) = 2(x - 8) + 10$

Expand the brackets: $6x - 18 = 2x - 16 + 10$
Simplify: $6x - 18 = 2x - 6$
Collect x terms on one side and numbers on the other: $4x = 12$
$x = 3$
Answer: x = 3

3 Inverse proportion

11 pumps take 12 hours to empty a pond.
All the pumps work at the same rate.
How many pumps are needed to empty a pond in 22 hours?

Total work = $11 \times 12 = 132$ pump-hour
Number needed = $132 \div 22 = 6$
Answer: 6

4 Angles in parallel lines

The diagram shows two parallel lines crossed by a straight line.
(a) Work out the value of x.
(b) Work out the size of the angle marked $(x + 10)°$.
The diagram is not drawn accurately.

(a) Corresponding angles are equal
$x + 10 = 4x - 59$
$3x = 69$
$x = 23$
(b) $1 \times 23 + 10 = 33$
Answer: (a) x = 23 (b) 33°

5 Rounding

(a) Write 0.02631 correct to 3 significant figures.
(b) Write 5.4362 correct to 2 decimal places.

(a) Count 3 significant figures and look at the next digit: 0.02631 rounds to 0.0263
(b) Look at the digit after decimal place 2: 5.4362 rounds to 5.44
Answer: (a) 0.0263 (b) 5.44

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6 Enlargements and coordinates

Shape A has an area of 6 cm^2.

Shape A is enlarged by scale factor $\frac{1}{2}$ to give shape B.

Work out the area of shape B.

Area scale factor = $(\frac{1}{2})^2 = \frac{1}{4}$

Area of B = $6 \times \frac{1}{4} = 1.5$

Answer: 1.5 cm^2

7 Average speed

Tom cycles 210 km at 70 km/h and then a further 210 km at 105 km/h.
Work out the average speed for the whole journey.

Time for first part = 210 ÷ 70 = 3 hours
Time for second part = 210 ÷ 105 = 2 hours
Total distance = 420 km, total time = 5 hours
Average speed = 420 ÷ 5 = 84 km/h
Answer: 84 km/h

8 Algebraic rearrangement and change of sign

(a) Show that the equation $x^3 + x - 23 = 0$ can be rearranged to give $x = \sqrt[3]{23 - x}$
(b) Show that the equation has a solution between x = 2 and x = 3

(a) Add 23 and subtract x from both sides: $x^3 = 23 - x$
Take the cube root of both sides: $x = \sqrt[3]{23 - x}$
(b) Let $f(x) = x^3 + x - 23$
f(2) = −13
f(3) = 7
There is a change of sign (negative to positive) and f is continuous, so there is a solution between x = 2 and x = 3
Answer: Shown: f(2) = −13, f(3) = 7 (change of sign)

9 Bounds and accuracy

A cyclist travels 560 metres in 18.1 seconds. The distance is correct to the nearest 10 metres and the time to the nearest 0.1 second.
(a) Work out the upper bound and the lower bound of the speed in m/s.
(b) Write the speed to a suitable degree of accuracy. Give a reason for your answer.

(a) Upper bound = 565 ÷ 18.05 = 31.3019
Lower bound = 555 ÷ 18.15 = 30.5785
(b) Both bounds round to 31 to 2 s.f. but not to the same value to 3 s.f.
So the speed is 31 m/s to 2 significant figures.
Answer: (a) 31.3019, 30.5785 (b) 31 m/s (2 s.f.)

10 Circle theorems: angles in the same segment

A, B, C and D are points on a circle. AC and BD meet at E.
Angle BAC = 61° and angle ACD = 51°.
Work out the size of angle x (angle AEB).
Give reasons for your working.

Angles in the same segment are equal, so angle ABD = angle ACD = 51°
In triangle ABE: angle BAE = 61° and angle ABE = 51°
x = 180 − 61 − 51 = 68
Answer: x = 68°