Day 226 **A Tier 10-a-day: Model Solutions**

1 Exchange rates

£1 = €1.25.
(a) Change £180 to euros.
(b) Change €175 to pounds.

(a) 180 × 1.25 = 225
(b) 175 ÷ 1.25 = 140
Answer: (a) €225 (b) £140

2 Circle area and circumference

A semicircle has diameter 22 cm.
Work out, in terms of π,
(a) the perimeter of the semicircle,
(b) the area of the semicircle.

(a) Curved edge = $\frac{1}{2} \times \pi \times 22 = 11\pi$

Perimeter = $11\pi + 22$

(b) Area = $\frac{1}{2} \times \pi \times 11^2 = \frac{121}{2}\pi$

Answer: (a) $11\pi + 22$ cm (b) $\frac{121}{2}\pi$ cm^2

3 Forming and solving equations

The sum of 3 consecutive whole numbers is 72.
Work out the largest of the numbers.

Let the smallest number be n. The numbers are n, n + 1, n + 2
Sum = 3n + 3 = 72
3n = 69
n = 23
The numbers are 23, 24, 25
Answer: 25

4 Estimating the mean from grouped data

The table shows the times, in minutes, taken by 50 people to finish a race.
(a) Write down the class that contains the median.
(b) Work out an estimate for the mean.

(a) The median is in position (50 + 1) ÷ 2 = 25.5. Cumulative frequencies: 19, 33, 42, 50
The median is in the class $40 < t \leq 60$
(b) Midpoints: 30, 50, 70, 90
30 × 19 + 50 × 14 + 70 × 9 + 90 × 8 = 2620
Estimate = 2620 ÷ 50 = 52.4
Answer: (a) $40 < t \leq 60$ (b) 52.4

5 HCF and LCM in index form

A = $2^4 \times 3 \times 7^4$
B = $2^2 \times 3^2 \times 7^3$
Find the highest common factor (HCF) and the lowest common multiple (LCM) of A and B.
Give your answers as products of prime factors.

HCF: for each prime take the lower power (a missing prime has power 0): $2^2 \times 3 \times 7^3$
LCM: for each prime take the higher power: $2^4 \times 3^2 \times 7^4$
Answer: HCF = $2^2 \times 3 \times 7^3$, LCM = $2^4 \times 3^2 \times 7^4$

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6 Volume of a prism

The diagram shows a prism.
The cross-section of the prism is a right-angled triangle.
Work out the volume of the prism.
The diagram is not drawn accurately.

Area of cross-section = $\frac{1}{2} \times 14 \times 6 = 42$

Volume = area of cross-section × length
= 42 × 7
= 294

Answer: 294 cm^3

7 Income tax

No tax is paid on the first £10,000 of income. Tax of 20% is paid on the next £35,000. Tax of 40% is paid on all income above £45,000. (Calculator)
Chloe earns £60,500 a year.
Work out the income tax paid.

Next £35,000 at 20%: £35,000 × 0.2 = £7,000
Above £45,000: £60,500 − £45,000 = £15,500, taxed at 40%: £15,500 × 0.4 = £6,200
Total tax = £7,000 + £6,200 = £13,200
Answer: £13,200

8 Bearings and distances

A ship leaves a port and travels 16 km on a bearing of 075°. (Calculator)
It then travels 12 km on a bearing of 345°.
(a) Work out how far it is now from the port.
(b) Work out the bearing of its final position from the port. Give your answer to the nearest degree.

The two bearings differ by 90°, so the path makes a right angle
(a) Distance^2 = 16^2 + 12^2 = 400, so distance = 20 km

(b) The angle at the port between the first leg and the line to the end: $\tan^{-1}(\frac{12}{16}) = 36.9°$

Bearing = 75° − 36.9° = 38.1°
Bearing = 038°
Answer: (a) 20 km (b) 038°

9 Probability without replacement

A bag contains 3 yellow and 7 white balls.
Two balls are taken at random without replacement.
Work out the probability that the two balls are the same colour.

P(yellow, yellow) = $\frac{3}{10} \times \frac{2}{9} = \frac{6}{90}$

P(white, white) = $\frac{7}{10} \times \frac{6}{9} = \frac{42}{90}$

Add: $\frac{6}{90} + \frac{42}{90} = \frac{8}{15}$

Answer: $\frac{8}{15}$

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10 Rearranging formulae (subject twice)

Make x the subject of $y = \frac{2x - 4}{2x - 2}$

Multiply both sides by $(2x - 2)$: $y(2x - 2) = 2x - 4$
Expand the bracket: $2xy - 2y = 2x - 4$
Collect the x terms on one side: $2xy - 2x = 2y - 4$
Factorise: $x(2y - 2) = 2y - 4$
Divide by $(2y - 2)$: $x = \frac{2y - 4}{2y - 2}$

Answer: $x = \frac{2y - 4}{2y - 2}$