Day 225 **A Tier 10-a-day: Model Solutions**

1 Primes

(a) Write down all the prime numbers between 30 and 46.
(b) Write 57 as the product of two prime numbers.

(a) A prime number has exactly two factors. Checking each number between 30 and 46 gives: 31, 37, 41, 43
(b) 57 ÷ 3 = 19, and both 3 and 19 are prime.
Answer: (a) 31, 37, 41, 43 (b) 3 × 19

2 Angles in regular polygons around a point

A regular octagon and an equilateral triangle share the side AB and are on opposite sides of it.
The two shapes meet at the point A.
Work out the size of the angle at A that is not inside either shape.

Interior angle of the regular octagon = 180 − 360 ÷ 8 = 135°
Interior angle of the equilateral triangle = 180 − 360 ÷ 3 = 60°
Angles round a point add up to 360°
360 − 135 − 60 = 165
Answer: 165°

3 Difference of two squares

(a) Factorise $9y^2 - 16z^2$
(b) Use the same idea to work out $7^2 - 3^2$ without a calculator.

(a) $9y^2 - 16z^2 = (3y)^2 - (4z)^2$
$(3y - 4z)(3y + 4z)$
(b) $a^2 - b^2 = (a + b)(a - b)$, so $7^2 - 3^2 = (7 + 3)(7 - 3)$
$= 10 \times 4 = 40$
Answer: (a) $(3y - 4z)(3y + 4z)$ (b) 40

4 Wages and overtime

Isla is paid £15.60 per hour for a basic week of 36 hours. (Calculator)
Overtime is paid at double time.
Last week Isla worked 7 hours of overtime.
Work out Isla's total pay for the week.

Basic pay = 36 × £15.60 = £561.60
Overtime rate = £15.60 × 2 = £31.20 per hour
Overtime pay = 7 × £31.20 = £218.40
Total = £561.60 + £218.40 = £780.00
Answer: £780.00

5 Rearranging to solve quadratics

Solve $2x^2 + x = 15$

Move everything to one side: $2x^2 + x - 15 = 0$
Factorise: $(x + 3)(2x - 5) = 0$
$x = -3$ or $x = \frac{5}{2}$
Answer: $x = -3$ or $x = \frac{5}{2}$

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6 Changing a ratio

A bag contains red counters and blue counters in the ratio 7 : 5.
18 more red counters are added to the bag.
The ratio of red counters to blue counters is now 9 : 5.
How many counters were in the bag at the start?

Let red = 7x and blue = 5x. After adding: (7x + 18) : 5x = 9 : 5

$\frac{7x + 18}{5x} = \frac{9}{5}$, so $5(7x + 18) = 9 \times 5x$

$35x + 90 = 45x$, so $10x = 90$, $x = 9$

Start: $7x + 5x = 12 \times 9 = 108$ counters

Answer: 108

7 Transformations of graphs

The graph of y = f(x) has a maximum point at (3, −3).
Write down the coordinates of the turning point of the graph of
(a) y = f(x) + 3
(b) y = 3f(x)

(a) Adding 3 to f(x) moves the graph 3 units up
The maximum point is (3, 0)
(b) Multiplying f(x) by 3 stretches the graph vertically by a factor of 3, so the y-coordinate is multiplied by 3
The maximum point is (3, −9)
Answer: (a) (3, 0) (b) (3, −9)

8 Bearings and distances

A boat leaves a port and travels 48 miles on a bearing of 185°. (Calculator)
It then travels 14 miles on a bearing of 095°.
(a) Work out how far it is now from the port.
(b) Work out the bearing of its final position from the port. Give your answer to the nearest degree.

The two bearings differ by 90°, so the path makes a right angle

(a) $\text{Distance}^2 = 48^2 + 14^2 = 2500$, so distance = 50 miles

(b) The angle at the port between the first leg and the line to the end: $\tan^{-1}(\frac{14}{48}) = 16.3°$

Bearing = 185° − 16.3° = 168.7°
Bearing = 169°
Answer: (a) 50 miles (b) 169°

9 Bounds in measurement

The length and width of a rectangular poster are 6.7 cm and 9.2 cm.
Each measurement is correct to 1 decimal place.
Work out the lower bound for the area.

Lower bound of length = 6.7 cm, lower bound of width = 9.2 cm
Lower bound of area = 6.65 × 9.15 = 60.8475
Answer: 60.8475 cm^2

10 Probability with three selections

A bag contains 4 red counters and 3 blue counters.
Three counters are taken at random without replacement.
Work out the probability that all three counters are red.

P(all red) = $\frac{4}{7} \times \frac{3}{6} \times \frac{2}{5}$

= $\frac{24}{210} = \frac{4}{35}$

Answer: $\frac{4}{35}$