Day 218 **A Tier 10-a-day: Model Solutions**

1 Sectors: finding an angle or radius

A sector has angle 60° at the centre and arc length 4π cm.
Work out the radius of the sector.

Arc length = $\frac{60}{360} \times 2 \times \pi \times r$

$4\pi = \frac{1}{3} \times r \times \pi$

$r = 4 \div \frac{1}{3}$

$r = 12$

Answer: 12 cm

2 Inequalities

Solve $-3x + 6 \geq x + 2$

Collect the x terms on the left: $-4x + 6 \geq 2$
Subtract 6: $-4x \geq -4$
Divide by -4. Dividing by a negative number reverses the inequality sign
$x \leq 1$
Answer: $x \leq 1$

3 Circle theorems: angle in a semicircle

PQ is a diameter of the circle.
Angle QPR = 31°.
Work out the size of angle x.
Give a reason for each step of your working.

The angle in a semicircle is 90°, so angle PRQ = 90°
Angles in a triangle add up to 180°
$x = 180 - 90 - 31 = 59$
Answer: x = 59°

4 Probabilities adding to 1

The table shows the probabilities of the outcomes of a biased spinner.
(a) Work out the value of x.
(b) The spinner is spun 100 times. Estimate the number of times it lands on green.

(a) Probabilities add to 1: $0.1 + x + 2x + 0.75 = 1$
$0.85 + 3x = 1$, so $3x = 0.15$
$x = 0.05$
(b) P(green) = 0.1, so expected number = $0.1 \times 100 = 10$
Answer: (a) x = 0.05 (b) 10

5 Perimeter and area with algebra

A rectangle has length $(2x - 3)$ cm and width $(2x - 4)$ cm.
The perimeter of the rectangle is 50 cm.
(a) Work out the value of x.
(b) Work out the area of the rectangle.

(a) $2 \times [(2x - 3) + (2x - 4)] = 50$
$(2x - 3) + (2x - 4) = 25$
$4x - 7 = 25$
$4x = 32$
$x = 8$
(b) Length = 13 cm and width = 12 cm
Area = $13 \times 12 = 156$
Answer: (a) x = 8 (b) 156 cm^2

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6 Growth and decay

The population of a town is 59,000 and increases by 20% per year. (Calculator)
Work out the population after 6 years.
Give your answer to the nearest whole number.

Multiplier = 1.2

$59{,}000 \times 1.2^6 = 176{,}173.06$

= 176,173 to the nearest whole number

Answer: 176,173

7 Expanding three brackets

Expand and simplify
$(x + 2)(2x - 1)(x - 5)$

Expand the first two brackets: $(x + 2)(2x - 1) = 2x^2 + 3x - 2$

Multiply by the third bracket: $(2x^2 + 3x - 2)(x - 5)$

$= 2x^3 + 3x^2 - 2x - 10x^2 - 15x + 10$

$= 2x^3 - 7x^2 - 17x + 10$

Answer: $2x^3 - 7x^2 - 17x + 10$

8 Recurring decimals, show that

(a) Show that $0.\dot{9}\dot{2} = \frac{92}{99}$

(b) Hence write $2.\dot{9}\dot{2}$ as a mixed number in its simplest form.

(a) Let x = 0.929292..., so 100x = 92.9292...

$100x - x = 92$, so $99x = 92$ and $x = \frac{92}{99}$

(b) $2.\dot{9}\dot{2} = 2 + \frac{92}{99} = 2\frac{92}{99}$

Answer: (b) $2\frac{92}{99}$

9 Negative and fractional indices

Work out the value of

(a) $100^{\frac{-1}{2}}$

(b) 2^{-4}

(a) $100^{\frac{-1}{2}} = \frac{1}{100^{\frac{1}{2}}} = \frac{1}{(\sqrt{100})} = \frac{1}{10}$

(b) $2^{-4} = \frac{1}{2^4} = \frac{1}{16}$

Answer: (a) $\frac{1}{10}$ (b) $\frac{1}{16}$

10 Probability with three selections

A bag contains 5 black pens and 4 blue pens.
Three pens are taken at random without replacement.
Work out the probability that exactly one of the three pens is blue.

One order, for example blue, black, black: $\frac{4}{9} \times \frac{5}{8} \times \frac{4}{7} = \frac{10}{63}$

There are 3 possible orders, each with the same probability

$3 \times \frac{10}{63} = \frac{10}{21}$

Answer: $\frac{10}{21}$