Day 214 **A Tier 10-a-day: Model Solutions**

1 Primes

(a) Write down all the prime numbers between 70 and 86.
(b) Write 209 as the product of two prime numbers.

(a) A prime number has exactly two factors. Checking each number between 70 and 86 gives: 71, 73, 79, 83
(b) 209 ÷ 11 = 19, and both 11 and 19 are prime.
Answer: (a) 71, 73, 79, 83 (b) 11 × 19

2 Laws of indices

Simplify fully

(a) $\frac{10x^7y^6}{3x^4y^5}$

(b) $\frac{(5x^3)^2}{2x^4}$

(a) Divide the numbers: 10 ÷ 3 = $\frac{10}{3}$

Subtract the powers: $x^7 \div x^4 = x^3$ and $y^6 \div y^5 = y$

= $\frac{10x^3y}{3}$

(b) Square the top: $(5x^3)^2 = 25x^6$

Divide the numbers: 25 ÷ 2 = $\frac{25}{2}$. Subtract the powers of x: 6 − 4 = 2

= $\frac{25x^2}{2}$

Answer: (a) $\frac{10x^3y}{3}$ (b) $\frac{25x^2}{2}$

3 Similar shapes: area and volume

Two similar models are 10 cm and 25 cm tall.
The surface area of the smaller model is 80 cm^2.
Work out the surface area of the larger model.

Length scale factor = 25 ÷ 10 = $\frac{5}{2}$

Area scale factor = $(\frac{5}{2})^2 = \frac{25}{4}$

Area = $80 \times \frac{25}{4} = 500$

Answer: 500 cm^2

4 Probabilities adding to 1

The table shows the probabilities of the outcomes of a biased spinner.
(a) Work out the value of x.
(b) The spinner is spun 300 times. Estimate the number of times it lands on green.

(a) Probabilities add to 1: 0.3 + x + 3x + 0.5 = 1
0.8 + 4x = 1, so 4x = 0.2
x = 0.05
(b) P(green) = 0.15, so expected number = 0.15 × 300 = 45
Answer: (a) x = 0.05 (b) 45

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5 Simultaneous equations

Solve the simultaneous equations
$2x - 2y = 0$
$-3x - 4y = -28$

Multiply (1) by 2 to make the y coefficients match
$4x - 4y = 0$ (1)
$-3x - 4y = -28$ (2)
The y terms have the same sign, so subtract equation (2) from equation (1)
$7x = 28$, so $x = 4$
Substitute $x = 4$ into equation (1) to find $y = 4$
Answer: x = 4, y = 4

6 Compound shapes with circles

The diagram shows a shape made from a rectangle and a semicircle.
Work out the perimeter of the shape.
Give your answer in terms of π.
The diagram is not drawn accurately.

Curved edge: $\frac{1}{2} \times \pi \times 4 = 2\pi$
Straight edges: $4 + 7 + 7 = 18$
Total = $18 + 2\pi$
Answer: 18 + 2π cm

7 Direct proportion formulae

y is directly proportional to the cube of x.
When x = 4, y = 320.
(a) Find a formula for y in terms of x.
(b) Work out the positive value of x when y = 135.

(a) $y = kx^3$. Substitute: $320 = k \times 64$, so $k = 5$
$y = 5x^3$
(b) $135 = 5x^3$, so $x^3 = 135 \div 5 = 27$, so $x = 3$
Answer: (a) $y = 5x^3$ (b) x = 3

8 Recurring decimals, show that

(a) Show that $0.\dot{9}\dot{0} = \frac{10}{11}$

(b) Hence write $2.\dot{9}\dot{0}$ as a mixed number in its simplest form.

(a) Let $x = 0.909090...$, so $100x = 90.9090...$
$100x - x = 90$, so $99x = 90$ and $x = \frac{90}{99} = \frac{10}{11}$

(b) $2.\dot{9}\dot{0} = 2 + \frac{10}{11} = 2\frac{10}{11}$

Answer: (b) $2\frac{10}{11}$

9 Finding an interest rate

Cara invests £2500 in a savings account. (Calculator)
The account pays compound interest at a fixed rate each year.
After 2 years there is £3025 in the account.
Work out the annual rate of interest.

Let the yearly multiplier be m. Then $2500 \times m^2 = 3025$
$m^2 = 3025 \div 2500 = 1.21$
$m = \sqrt{1.21} = 1.1$
Rate = $(1.1 - 1) \times 100 = 10\%$
Answer: 10%

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10 Circle theorems: cyclic quadrilaterals

A, B, C and D are points on a circle.
Angle B = (3x + 25)° and angle D = (2x + 45)°.
(a) Work out the value of x.
(b) Work out the size of angle B.
The diagram is not drawn accurately.

(a) Opposite angles of a cyclic quadrilateral add up to 180°
3x + 25 + 2x + 45 = 180
5x + 70 = 180
5x = 110
x = 22
(b) 3 × 22 + 25 = 91
Answer: (a) x = 22 (b) 91°