Day 213 **A Tier 10-a-day: Model Solutions**

1 Expanding double brackets

Expand and simplify

$(x + 4)^2$

Multiply every term in the first bracket by every term in the second

$= (x + 4)(x + 4) = x^2 + 4x + 4x + 16$

$= x^2 + 8x + 16$

Answer: $x^2 + 8x + 16$

2 Circle area and circumference

A circular table top has radius 14 cm.
Work out, in terms of π,
(a) the circumference,
(b) the area.

(a) Circumference = $\pi d = \pi \times 28 = 28\pi$

(b) Area = $\pi r^2 = \pi \times 14^2 = 196\pi$

Answer: (a) 28π cm (b) 196π cm^2

3 Converting units

(a) Change 8 cm^2 to mm^2.
(b) Change 8 litres to cm^3.

(a) 8 cm^2 × 100 = 800 mm^2

(b) 8 litres × 1,000 = 8,000 cm^3

Answer: (a) 800 mm^2 (b) 8,000 cm^3

4 Angles in regular polygons around a point

A regular decagon and a regular pentagon share the side AB and are on opposite sides of it.
The two shapes meet at the point A.
Work out the size of the angle at A that is not inside either shape.

Interior angle of the regular decagon = 180 − 360 ÷ 10 = 144°

Interior angle of the regular pentagon = 180 − 360 ÷ 5 = 108°

Angles round a point add up to 360°

360 − 144 − 108 = 108

Answer: 108°

5 Compound interest

Leah invests £2000 in a savings account. (Calculator)
The account pays 4% compound interest per year.
Work out the interest earned in 4 years.

Multiplier = 100% + 4% = 1.04

£2000 × 1.04^4 = £2339.717

Interest = £2339.72 − £2000 = £339.72

Answer: £339.72

6 Volume of composite solids

A solid is made from a cylinder of radius 9 cm and a hemisphere on top, with the same radius.
The total volume of the solid is 810π cm^3.
Work out the height of the cylinder.

Hemisphere = $\frac{2}{3} \times \pi \times 9^3 = 486\pi$

Cylinder volume = $810\pi - 486\pi = 324\pi$

$\pi \times 9^2 \times h = 324\pi$

h = 324 ÷ 81 = 4

Answer: 4 cm

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7 Adding and subtracting in standard form

Work out $(4.8 \times 10^{-1}) + (9.1 \times 10^{-3})$
Give your answer in standard form.

Write both numbers using 10^{-3}: $4.8 \times 10^{-1} = 480 \times 10^{-3}$
$(480 + 9.1) \times 10^{-3} = 489.1 \times 10^{-3}$
In standard form: 4.891×10^{-1}
Answer: 4.891×10^{-1}

8 Algebraic rearrangement and change of sign

(a) Show that the equation $x^3 + 5x - 14 = 0$ can be rearranged to give $x = \sqrt[3]{14 - 5x}$
(b) Show that the equation has a solution between $x = 1$ and $x = 2$

(a) Add 14 and subtract 5x from both sides: $x^3 = 14 - 5x$
Take the cube root of both sides: $x = \sqrt[3]{14 - 5x}$
(b) Let $f(x) = x^3 + 5x - 14$
$f(1) = -8$
$f(2) = 4$
There is a change of sign (negative to positive) and f is continuous, so there is a solution between $x = 1$ and $x = 2$
Answer: Shown: $f(1) = -8$, $f(2) = 4$ (change of sign)

9 Rationalising with conjugates

Rationalise the denominator of $\frac{3}{2 - \sqrt{3}}$

Give your answer in the form $a + b\sqrt{c}$ where a and b are integers.

Multiply top and bottom by the conjugate $2 + \sqrt{3}$: $\frac{3(2 + \sqrt{3})}{(2 - \sqrt{3})(2 + \sqrt{3})}$

Denominator: $4 - 3 = 1$
$\frac{3(2 + \sqrt{3})}{1} = 3(2 + \sqrt{3})$
$= 6 + 3\sqrt{3}$
Answer: $6 + 3\sqrt{3}$

10 Finding an interest rate

Cara invests £8000 in a savings account. (Calculator)
The account pays compound interest at a fixed rate each year.
After 2 years there is £8652.80 in the account.
Work out the annual rate of interest.

Let the yearly multiplier be m. Then $8000 \times m^2 = 8652.8$
$m^2 = 8652.8 \div 8000 = 1.0816$
$m = \sqrt{1.0816} = 1.04$
Rate $= (1.04 - 1) \times 100 = 4\%$
Answer: 4%