Day 212 **A Tier 10-a-day: Model Solutions**

1 Laws of indices

Simplify fully

(a) $\frac{18x^4y^3}{5xy^6}$

(b) $\frac{(5x^2)^2}{3x^3}$

(a) Divide the numbers: $18 \div 5 = \frac{18}{5}$

Subtract the powers: $x^4 \div x = x^3$ and $y^3 \div y^6 = y^{-3}$

$= \frac{18x^3}{5y^3}$

(b) Square the top: $(5x^2)^2 = 25x^4$

Divide the numbers: $25 \div 3 = \frac{25}{3}$. Subtract the powers of x: $4 - 3 = 1$

$= \frac{25x}{3}$

Answer: (a) $\frac{18x^3}{5y^3}$ (b) $\frac{25x}{3}$

2 Map scales

The scale of a map is 1 : 200000.
A road is 2 km long.
Work out the length of the road on the map, in centimetres.

2 km = 200000 cm
Length on map = 200000 ÷ 200000 = 1 cm
Answer: 1 cm

3 Profit and loss

Jack sells a phone for £308 and makes a profit of 10%.
Work out how much Jack paid for the phone.

Selling price = 110% of the cost price
110% = £308, so 1% = £2.8
100% = £280
Answer: £280

4 Surface area of a cuboid

A cuboid has length 5 cm and width 2 cm.
Its total surface area is 146 cm^2.
Work out the height of the cuboid.

$2 \times (5 \times 2 + 5\,h + 2\,h) = 146$
$10 + 7h = 73$
$7h = 63$
$h = 63 \div 7 = 9$
Answer: 9 cm

5 Income tax

No tax is paid on the first £11,000 of income. Tax of 20% is paid on the next £28,000. Tax of 40% is paid on all income above £39,000. (Calculator)
Sara earns £50,500 a year.
Work out the income tax paid.

Next £28,000 at 20%: £28,000 × 0.2 = £5,600
Above £39,000: £50,500 − £39,000 = £11,500, taxed at 40%: £11,500 × 0.4 = £4,600
Total tax = £5,600 + £4,600 = £10,200
Answer: £10,200

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6 Surface area of a prism

The diagram shows a triangular prism.
The cross-section is a right-angled triangle.
Work out the total surface area of the prism.
The diagram is not drawn accurately.

Hypotenuse = $\sqrt{7^2 + 24^2}$ = 25 cm

Two triangular ends: $2 \times \frac{1}{2} \times 7 \times 24 = 168$

Three rectangles: $(7 + 24 + 25) \times 20 = 1120$

Total = 168 + 1120 = 1288

Answer: 1288 cm^2

7 Stratified sampling

A club has 50 members in junior, 90 in intermediate and 100 in senior.
A stratified sample of 24 members is to be taken.
Work out how many members should be chosen from intermediate.

Total = 50 + 90 + 100 = 240

Number from intermediate = $\frac{90}{240} \times 24$

= 9

Answer: 9

8 Angles in regular polygons around a point

A regular hexagon and a regular pentagon share the side PQ and are on opposite sides of it.
The two shapes meet at the point P.
Work out the size of the angle at P that is not inside either shape.

Interior angle of the regular hexagon = 180 − 360 ÷ 6 = 120°
Interior angle of the regular pentagon = 180 − 360 ÷ 5 = 108°
Angles round a point add up to 360°
360 − 120 − 108 = 132
Answer: 132°

9 Transformations of graphs

The graph of y = f(x) has a maximum point at (5, 3).
Write down the coordinates of the turning point of the graph of
(a) y = 3f(x)
(b) y = f(−x)

(a) Multiplying f(x) by 3 stretches the graph vertically by a factor of 3, so the y-coordinate is multiplied by 3
The maximum point is (5, 9)
(b) The graph is reflected in the y-axis, so the x-coordinate changes sign
The maximum point is (−5, 3)
Answer: (a) (5, 9) (b) (−5, 3)

10 Recurring decimals (mixed)

Write $0.9\dot{7}$ as a fraction in its simplest form.

Let x = 0.977...
Multiply by 10: 10x = 9.77...
Multiply by 100: 100x = 97.77...
Subtract: 90x = 97 − 9 = 88

$x = \frac{88}{90} = \frac{44}{45}$

Answer: $\frac{44}{45}$