Day 21 **A Tier 10-a-day: Model Solutions**

1 Primes

(a) Write down all the prime numbers between 90 and 104.
(b) Write 95 as the product of two prime numbers.

(a) A prime number has exactly two factors. Checking each number between 90 and 104 gives: 97, 101, 103
(b) $95 \div 5 = 19$, and both 5 and 19 are prime.
Answer: (a) 97, 101, 103 (b) 5×19

2 Simultaneous equations

Solve the simultaneous equations
$-5x + 3y = -37$
$-2x + 2y = -18$

Multiply (1) by 2 and (2) by 3 to make the y coefficients match
$-10x + 6y = -74$ (1)
$-6x + 6y = -54$ (2)
The y terms have the same sign, so subtract equation (1) from equation (2)
$4x = 20$, so $x = 5$
Substitute $x = 5$ into equation (1) to find $y = -4$
Answer: $x = 5$, $y = -4$

3 Volume of a cylinder

A cylinder has radius 10 cm and volume 700π cm^3.
Work out the height of the cylinder.

$\pi r^2 h = 700\pi$
$100\pi \times h = 700\pi$
$h = 700 \div 100 = 7$
Answer: 7 cm

4 Sharing in a ratio

Isla and Jack share 90 football cards in the ratio 8 : 1.
How many more football cards does Isla get than Jack?

Total number of parts = 8 + 1 = 9
One part = 90 ÷ 9 = 10
Isla gets 8 × 10 = 80 and Jack gets 1 × 10 = 10
Difference = 80 − 10 = 70
Answer: 70

5 Income tax

No tax is paid on the first £12,500 of income. Tax of 20% is paid on the next £28,000. Tax of 45% is paid on all income above £40,500. (Calculator)
Aisha earns £57,000 a year.
Work out the income tax paid.

Next £28,000 at 20%: £28,000 × 0.2 = £5,600
Above £40,500: £57,000 − £40,500 = £16,500, taxed at 45%: £16,500 × 0.45 = £7,425
Total tax = £5,600 + £7,425 = £13,025
Answer: £13,025

6 Probability without replacement

A bag contains 2 red, 3 blue and 5 green counters.
Two counters are taken at random without replacement.
Work out the probability that the two counters are different colours.

$$\text{P(same colour)} = \frac{2}{10} \times \frac{1}{9} + \frac{3}{10} \times \frac{2}{9} + \frac{5}{10} \times \frac{4}{9} = \frac{14}{45}$$

$$\text{P(different)} = 1 - \frac{14}{45} = \frac{31}{45}$$

Answer: $\frac{31}{45}$

7 Forming quadratic equations

A rectangle has width x cm and length (x + 1) cm.

The area of the rectangle is 90 cm^2.

(a) Show that $x^2 + 1x - 90 = 0$

(b) Work out the value of x.

(a) Area = length × width, so $x(x + 1) = 90$

$x^2 + 1x = 90$

$x^2 + 1x - 90 = 0$

(b) Factorise: $(x - 9)(x + 10) = 0$

$x = 9$ or $x = -10$

A length cannot be negative, so $x = 9$

Answer: (a) Shown (b) x = 9

8 Laws of indices

Write $5^3 \times (5^7)^6$ as a single power of 5.

$(5^7)^6 = 5^{42}$

Add the powers: $3 + 42 = 45$

$= 5^{45}$

Answer: 5^{45}

9 Circle theorems: cyclic quadrilaterals

A, B, C and D are points on a circle.

Angle A = (3x + 15)° and angle C = (4x + 25)°.

(a) Work out the value of x.

(b) Work out the size of angle A.

The diagram is not drawn accurately.

(a) Opposite angles of a cyclic quadrilateral add up to 180°

$3x + 15 + 4x + 25 = 180$

$7x + 40 = 180$

$7x = 140$

$x = 20$

(b) $3 \times 20 + 15 = 75$

Answer: (a) x = 20 (b) 75°

10 Simultaneous equations (one quadratic)

Solve the simultaneous equations

$x^2 + y^2 = 100$

$y = x + 2$

Substitute $y = x + 2$ into $x^2 + y^2 = 100$

$x^2 + (x + 2)^2 = 100$

$2x^2 + 4x - 96 = 0$

Divide by 2: $x^2 + 2x - 48 = 0$

$(x + 8)(x - 6) = 0$

$x = -8$ or $x = 6$

Use $y = x + 2$ to find y: when $x = -8$, $y = -6$; when $x = 6$, $y = 8$

Answer: x = −8, y = −6 and x = 6, y = 8