Day 205 **A Tier 10-a-day: Model Solutions**

1 Successive percentage changes

The value of a house is increased by 40% and then the new amount is decreased by 15%.
Work out the overall percentage increase.

Multipliers: increase of 40% is 1.4, decrease of 15% is 0.85
Overall multiplier = 1.4 × 0.85 = 1.19
1.19 means a percentage increase of 19%
Answer: 19% increase

2 Area of shaded regions with circles

2 identical circles of radius 2 cm fit exactly inside a rectangle.
The circles touch each other and the sides of the rectangle. (Calculator)
Work out the area of the shaded region.
Give your answer to 3 significant figures.

Rectangle: $8 \times 4 = 32$ cm^2
2 circles: $2 \times \pi \times 2^2 = 8\pi$
Shaded area = $32 - 8\pi = 6.87$
Answer: 6.87 cm^2

3 nth term of a linear sequence

Here are the first four terms of a sequence.
21, 24, 27, 30
(a) Find an expression for the nth term.
(b) Is 119 a term of this sequence? You must show your working.

The terms go up by 3 each time, so the nth term starts with 3n
$21 - 3 = 18$, so the nth term is 3n + 18
(b) 3n + 18 = 119
$3n = 101$, so $n = \frac{101}{3}$
n is not a positive whole number, so 119 is not a term of the sequence
Answer: (a) 3n + 18 (b) No

4 Profit and loss

Kiran buys a games console for £250 and sells it for £225.
Work out Kiran's percentage loss.

Loss = £250 − £225 = £25
Percentage loss = $\frac{25}{250} \times 100 = 10\%$
Answer: 10%

5 Capture-recapture

A scientist wants to estimate the number of deer in a forest.
She catches 100 deer, marks each one and returns them.
Later she catches a second sample of 50 deer. 5 of these are marked.
Work out an estimate for the total number of deer in a forest.

Proportion marked in the second sample = $\frac{5}{50}$
This is the same as the proportion in the whole population: $\frac{100}{N}$
$\frac{100}{N} = \frac{5}{50}$, so $N = 100 \times 50 \div 5 = 1000$
Answer: 1000

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6 Area of a sector

The diagram shows a sector of a circle of radius 15 cm.
Work out the area of the sector.
Give your answer in terms of π.
The diagram is not drawn accurately.

Area = $\frac{240}{360} \times \pi \times 15^2$

= $\frac{2}{3} \times 225\,\pi$

= 150π

Answer: 150π cm^2

7 Frequency density

The table gives information about 122 measurements.
A histogram is drawn using this information.
(a) Work out the missing frequency.
(b) Work out the height of the bar for the class $10 < h \leq 15$.

(a) Missing frequency = $122 - (7 + 66 + 40) = 9$
(b) Class width = $15 - 10 = 5$
Frequency density = $7 \div 5 = 1.4$
Answer: (a) 9 (b) 1.4

8 Expanding and simplifying

Expand and simplify
$(x - 6)^2 - (x + 4)(x + 6)$

$(x - 6)^2 = x^2 - 12x + 36$
$(x + 4)(x + 6) = x^2 + 10x + 24$
Subtract (take care with the signs): $(x^2 - 12x + 36) - (x^2 + 10x + 24)$
$= -22x + 12$
Answer: $-22x + 12$

9 Rationalising with conjugates

Rationalise the denominator of $\frac{2}{3 - \sqrt{7}}$

Give your answer in the form $a + b\sqrt{c}$ where a and b are integers.

Multiply top and bottom by the conjugate $3 + \sqrt{7}$: $\frac{2(3 + \sqrt{7})}{(3 - \sqrt{7})(3 + \sqrt{7})}$

Denominator: $9 - 7 = 2$
$\frac{2(3 + \sqrt{7})}{2} = 1(3 + \sqrt{7})$
$= 3 + \sqrt{7}$
Answer: $3 + \sqrt{7}$

10 Probability with three selections

A bag contains 3 red counters and 7 blue counters.
Three counters are taken at random without replacement.
Work out the probability that at least one of the three counters is blue.

P(none is blue) = P(all red) = $\frac{3}{10} \times \frac{2}{9} \times \frac{1}{8} = \frac{1}{120}$

P(at least one blue) = $1 - \frac{1}{120} = \frac{119}{120}$

Answer: $\frac{119}{120}$