Day 204 **A Tier 10-a-day: Model Solutions**

1 Finding a percentage change

The number of students in a school was 300 and is now 180.
Work out the percentage decrease.

Change = 300 − 180 = 120

Percentage decrease = $\frac{120}{300} \times 100 = 40\%$

Answer: 40% decrease

2 Sharing in a ratio

Hugo, Jack and Dev share some money in the ratio 2 : 1 : 5.
Hugo gets £6.
How much does Dev get?

2 parts = £6, so 1 part = £6 ÷ 2 = £3
Dev gets 5 parts: 5 × £3 = £15
Answer: £15

3 Difference of two squares

(a) Factorise $x^2 - 144$
(b) Use the same idea to work out $84^2 - 16^2$ without a calculator.

(a) Difference of two squares: $x^2 - 144 = x^2 - 12^2$
$(x - 12)(x + 12)$
(b) $a^2 - b^2 = (a + b)(a - b)$, so $84^2 - 16^2 = (84 + 16)(84 - 16)$
$= 100 \times 68 = 6800$
Answer: (a) $(x - 12)(x + 12)$ (b) 6800

4 Similar shapes: finding a length

The two triangles are similar.
Work out the value of x.
The diagrams are not drawn accurately.

Scale factor from the small triangle to the large triangle = 35 ÷ 28 = $\frac{5}{4}$

$x = 20 \times \frac{5}{4} = 25$

Answer: x = 25 m

5 Simultaneous equations

Solve the simultaneous equations
$6x + 3y = 0$
$-x - 2y = -9$

Multiply (1) by 2 and (2) by 3 to make the y coefficients match
$12x + 6y = 0$ (1)
$-3x - 6y = -27$ (2)
The y terms have opposite signs, so add the equations
$9x = -27$, so $x = -3$
Substitute $x = -3$ into equation (1) to find $y = 6$
Answer: x = −3, y = 6

6 Addition and multiplication rules

For two events A and B:
P(A) = 0.6, P(B) = 0.6 and P(A or B) = 0.85.
(a) Work out P(A and B).
(b) Work out the probability that neither A nor B happens.

(a) P(A and B) = P(A) + P(B) − P(A or B) = 0.6 + 0.6 − 0.85 = 0.35
(b) 1 − P(A or B) = 1 − 0.85 = 0.15
Answer: (a) 0.35 (b) 0.15

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7 Recurring decimals, show that

(a) Show that $0.\dot{6}\dot{8} = \frac{68}{99}$

(b) Hence write $5.\dot{6}\dot{8}$ as a mixed number in its simplest form.

(a) Let x = 0.686868..., so 100x = 68.6868...

$100x - x = 68$, so $99x = 68$ and $x = \frac{68}{99}$

(b) $5.\dot{6}\dot{8} = 5 + \frac{68}{99} = 5\frac{68}{99}$

Answer: (b) $5\frac{68}{99}$

8 Bearings with the cosine rule

Ship A sails 21 km from a port on a bearing of 270°.
Ship B sails 10 km from the same port on a bearing of 010°. (Calculator)
Work out the distance between the two ships.
Give your answer to 3 significant figures.

The angle between the two paths = $360° - |270° - 10°| = 100°$

$d^2 = 21^2 + 10^2 - 2 \times 21 \times 10 \times \cos 100°$

$d^2 = 613.93$

$d = 24.8$

Answer: 24.8 km

9 Surface area of a prism

The diagram shows a triangular prism.
The cross-section is a right-angled triangle.
Work out the total surface area of the prism.
The diagram is not drawn accurately.

Hypotenuse = $\sqrt{8^2 + 6^2} = 10$ cm

Two triangular ends: $2 \times \frac{1}{2} \times 8 \times 6 = 48$

Three rectangles: $(8 + 6 + 10) \times 4 = 96$

Total = 48 + 96 = 144

Answer: 144 cm^2

10 Simultaneous equations (one quadratic)

Solve the simultaneous equations

$x^2 + y^2 = 25$

$y = -2x - 5$

Substitute $y = -2x - 5$ into $x^2 + y^2 = 25$

$x^2 + (-2x - 5)^2 = 25$

$5x^2 + 20x = 0$

Divide by 5: $x^2 + 4x = 0$

$(x + 4)(x + 0) = 0$

$x = -4$ or $x = 0$

Use $y = -2x - 5$ to find y: when $x = -4$, $y = 3$; when $x = 0$, $y = -5$

Answer: x = −4, y = 3 and x = 0, y = −5