Day 203 **A Tier 10-a-day: Model Solutions**

1 Converting units of area and volume

Convert 250000 cm^2 into m^2.

1 m^2 = 10 000 cm^2
250000 ÷ 10 000 = 25
Answer: 25 m^2

2 Percentage change

The number of visitors to a museum changes from 500 to 400.
Work out the percentage decrease.

Decrease = 500 − 400 = 100
Percentage decrease = $\frac{100}{500} \times 100 = 20\%$
Answer: 20%

3 Volume of a pyramid

The diagram shows a pyramid with a square base of side 9 cm.
The volume of the pyramid is 270 cm^3.
Work out the perpendicular height, h, of the pyramid.
The diagram is not drawn accurately.

$\frac{1}{3} \times 9 \times 9 \times h = 270$
27 h = 270
h = 270 ÷ 27 = 10
Answer: h = 10

4 Powers and roots in standard form

Work out $(5 \times 10^{-6})^2$
Give your answer in standard form.

$(5 \times 10^{-6})^2 = 5^2 \times (10^{-6})^2$
$= 25 \times 10^{-12}$
$= 2.5 \times 10^{-11}$
Answer: 2.5×10^{-11}

5 Column vectors

$\mathbf{u} = \begin{pmatrix} -12 \\ 5 \end{pmatrix}$.
Work out the magnitude (length) of $\mathbf{u}$.

Magnitude = $\sqrt{(-12)^2 + (5)^2}$
$= \sqrt{144 + 25} = \sqrt{169}$
= 13
Answer: 13

6 Mixed number multiplication and division

Work out $4\frac{5}{7} \times 2\frac{1}{2}$
Give your answer as a mixed number in its simplest form.

Improper fractions: $\frac{33}{7}$ and $\frac{5}{2}$
Multiply: $\frac{33}{7} \times \frac{5}{2} = \frac{165}{14}$
Simplify: $11\frac{11}{14}$
Answer: $11\frac{11}{14}$

7 Circles and tangents

The point P(−3, 4) lies on a circle with centre O(0, 0).
(a) Work out the equation of the circle, in the form $x^2 + y^2 = r^2$.
(b) Find the equation of the tangent to the circle at P.

(a) The circle is $x^2 + y^2 = r^2$, so $r^2 = (-3)^2 + 4^2 = 25$

(b) Gradient of the radius OP = $\frac{4}{-3} = -\frac{4}{3}$

The tangent is perpendicular to the radius, so its gradient = $-1 \div (-\frac{4}{3}) = \frac{3}{4}$

$y = \frac{3}{4}x + c$ through (−3, 4): $4 = \frac{3}{4} \times (-3) + c$, so $c = \frac{25}{4}$

$y = \frac{3}{4}x + \frac{25}{4}$

Answer: (a) $x^2 + y^2 = 25$ (b) $y = \frac{3}{4}x + \frac{25}{4}$

8 Direct proportion formulae

y is directly proportional to the square root of x.
When x = 25, y = 25.
(a) Find a formula for y in terms of x.
(b) Work out the positive value of x when y = 5.

(a) $y = k\sqrt{x}$. Substitute: $25 = k \times 5$, so $k = 5$

$y = 5\sqrt{x}$

(b) $5 = 5\sqrt{x}$, so $\sqrt{x} = 5 \div 5 = 1$, so $x = 1^2 = 1$

Answer: (a) $y = 5\sqrt{x}$ (b) $x = 1$

9 Rationalising with conjugates

Rationalise the denominator of $\frac{20}{4 - \sqrt{6}}$

Give your answer in the form $a + b\sqrt{c}$ where a and b are integers.

Multiply top and bottom by the conjugate $4 + \sqrt{6}$: $\frac{20(4 + \sqrt{6})}{(4 - \sqrt{6})(4 + \sqrt{6})}$

Denominator: $16 - 6 = 10$

$\frac{20(4 + \sqrt{6})}{10} = 2(4 + \sqrt{6})$

$= 8 + 2\sqrt{6}$

Answer: $8 + 2\sqrt{6}$

10 Perpendicular bisector

A is the point (−7, 1) and B is the point (1, 5).
Find the equation of the perpendicular bisector of AB.

Midpoint of AB = $(\frac{(-7) + 1}{2}, \frac{1 + 5}{2}) = (-3, 3)$

Gradient of AB = $\frac{4}{8} = \frac{1}{2}$

Perpendicular gradient = $-1 \div \frac{1}{2} = -2$

$y = -2x + c$ through (−3, 3): $3 = -2 \times (-3) + c$, so $c = -3$

$y = -2x - 3$

Answer: $y = -2x - 3$