Day 201 **A Tier 10-a-day: Model Solutions**

1 Direct proportion

Here are the ingredients for 4 biscuits.
Water: 300 ml
Butter: 40 g
Milk: 120 ml
(a) Work out how much water is needed for 9 biscuits.
(b) Hannah has 285 g of butter. What is the greatest number of biscuits she can make?

(a) water for one: 300 ÷ 4 = 75 ml
For 9: 75 × 9 = 675 ml
(b) butter for one: 40 ÷ 4 = 10 g
285 ÷ 10 = 28 remainder 5, so she can make 28
Answer: (a) 675 ml (b) 28

2 Standard form notation

(a) Write 0.0089 in standard form.
(b) Write 1.25×10^5 as an ordinary number.

(a) Move the decimal point 3 places to get 8.9: 8.9×10^{-3}
(b) Move the decimal point 5 places to the right: 125,000
Answer: (a) 8.9×10^{-3} (b) 125,000

3 Probabilities adding to 1

The table shows the probabilities of the outcomes of a biased four-sided dice.
(a) Work out the value of x.
(b) The four-sided dice is rolled 200 times. Estimate the number of times it lands on 2.

(a) Probabilities add to 1: 0.3 + x + x + 0.2 = 1
0.5 + 2x = 1, so 2x = 0.5
x = 0.25
(b) P(2) = 0.25, so expected number = 0.25 × 200 = 50
Answer: (a) x = 0.25 (b) 50

4 Circle theorems: tangents

PA is a tangent to a circle with centre O. A is on the circle.
The radius is 9 cm and OP = 15 cm.
Work out the length of PA.
The diagram is not drawn accurately.

The tangent is perpendicular to the radius, so angle OAP = 90°
$PA^2 = 15^2 - 9^2 = 225 - 81 = 144$
$PA = \sqrt{144} = 12$
Answer: 12 cm

5 Composite functions

f(x) = 2x + 1
$g(x) = x^2 - 5$
Solve gf(x) = 20

$gf(x) = (2x + 1)^2 - 5$
$(2x + 1)^2 - 5 = 20$, so $(2x + 1)^2 = 25$
$2x + 1 = \pm 5$
2x + 1 = 5 or 2x + 1 = −5
x = 2 or x = −3
Answer: x = 2 or x = −3

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6 Fraction calculations

Work out $(4\frac{1}{3} - 1\frac{5}{6}) \times 1\frac{1}{4}$

Give your answer as a mixed number in its simplest form.

Brackets first, as improper fractions: $\frac{13}{3} - \frac{11}{6} = \frac{5}{2}$

Then multiply: $\frac{5}{2} \times \frac{5}{4} = \frac{25}{8}$

$= 3\frac{1}{8}$

Answer: $3\frac{1}{8}$

7 Simplifying algebraic fractions

Simplify fully

$$\frac{2x^2 + 5x - 3}{3x^2 + 13x + 12}$$

Factorise the top: $(2x - 1)(x + 3)$
Factorise the bottom: $(3x + 4)(x + 3)$
Cancel the common factor $(x + 3)$

Answer: $\frac{2x - 1}{3x + 4}$

8 Finding a rate of compound change

Priya invests £1,500 in a savings account. (Calculator)
Compound interest is paid at a fixed rate each year.
After 2 years the investment is worth £2,160.
Work out the annual rate of interest.

Overall multiplier = £2,160 ÷ £1,500 = 1.44
Take the square root: $\sqrt{1.44} = 1.2$
Rate = $(1.2 - 1) \times 100 = 20\%$
Answer: 20%

9 Equations with algebraic fractions

Solve $\frac{4}{(x + 1)} + \frac{1}{(x - 5)} = 1$

Multiply every term by $(x + 1)(x - 5)$
$4(x - 5) + 1(x + 1) = (x + 1)(x - 5)$
$5x - 19 = x^2 - 4x - 5$
$x^2 - 9x + 14 = 0$
$(x - 2)(x - 7) = 0$
$x = 2$ or $x = 7$
Answer: $x = 2$ or $x = 7$

10 The cosine rule: finding an angle

Work out the size of angle x. (Calculator)
Give your answer to 1 decimal place.
The diagram is not drawn accurately.

$$\cos x = \frac{15^2 + 19^2 - 9^2}{2 \times 15 \times 19}$$

$$\cos x = \frac{505}{570}$$

$x = \cos^{-1}(0.8860)$
$x = 27.6°$
Answer: $x = 27.6°$