Day 20 **A Tier 10-a-day: Model Solutions**

1 Loci and constructions

A radio mast is at point A. Its signal reaches up to 4 km.
A radio mast at B reaches up to 6 km.
Describe the region where both signals can be received.

Each signal covers the inside of a circle
Both signals are received where the two circles overlap
Answer: Inside both circles: the overlap of a circle of radius 4 km centred on A and a circle of radius 6 km centred on B

2 Mixed number addition and subtraction

Work out $1\frac{1}{2} - 1\frac{1}{6}$

Give your answer as a mixed number in its simplest form.

Write as improper fractions: $1\frac{1}{2} = \frac{3}{2}$ and $1\frac{1}{6} = \frac{7}{6}$

Common denominator 6: $\frac{9}{6} - \frac{7}{6}$

$= \frac{2}{6}$

$= \frac{1}{3}$

Answer: $\frac{1}{3}$

3 Surface area of a cuboid

A closed gift box is a cuboid measuring 12 cm by 10 cm by 2 cm.
Work out the total surface area.

Faces: 12 × 10, 12 × 2 and 10 × 2 each appear twice
2 × (120 + 24 + 20)
= 2 × 164 = 328
Answer: 328 cm^2

4 Scatter graphs

The scatter graph shows the hours of sunshine and the number of ice creams sold on some days.
A line of best fit is drawn.
(a) Describe the correlation.
(b) Estimate the number of ice creams sold on a day with 8 hours of sunshine.

(a) As hours of sunshine increases, ice creams sold increases, so there is positive correlation
(b) Draw a line up from 8 on the horizontal axis to the line of best fit, then across to the vertical axis
The line gives a value of 42
Answer: (a) Positive correlation (b) 42

5 Volume of a prism

The diagram shows a prism with a trapezium as its cross-section.
The volume of the prism is 539 cm^3.
Work out the value of x.
The diagram is not drawn accurately.

Area of cross-section = $\frac{1}{2} \times (3 + 8) \times 7 = 38.5$

Volume = area × length
539 = 38.5 × length
length = 539 ÷ 38.5 = 14
Answer: x = 14

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6 Index equations

Solve $2^{2x+3} = 2048$

$2048 = 2^{11}$, so $2x + 3 = 11$
$2x = 11 - 3 = 8$
$x = 4$
Answer: x = 4

7 Addition and multiplication rules

A and B are independent events.
P(A) = 0.7 and P(B) = 0.2.
(a) Work out P(A and B).
(b) Work out P(A or B).

(a) Independent, so multiply: $0.7 \times 0.2 = 0.14$
(b) P(A or B) = P(A) + P(B) − P(A and B) = 0.7 + 0.2 − 0.14 = 0.76
Answer: (a) 0.14 (b) 0.76

8 Algebraic rearrangement and change of sign

(a) Show that the equation $x^3 + 4x - 30 = 0$ can be rearranged to give $x = \sqrt[3]{30 - 4x}$
(b) Show that the equation has a solution between x = 2 and x = 3

(a) Add 30 and subtract 4x from both sides: $x^3 = 30 - 4x$
Take the cube root of both sides: $x = \sqrt[3]{30 - 4x}$
(b) Let $f(x) = x^3 + 4x - 30$
f(2) = −14
f(3) = 9
There is a change of sign (negative to positive) and f is continuous, so there is a solution between x = 2 and x = 3
Answer: Shown: f(2) = −14, f(3) = 9 (change of sign)

9 Reading a histogram

The histogram shows information about the age of some items.
(a) Work out the number of items with a age in the class 30 to 50 years.
(b) Work out an estimate for the number with a age greater than 15 years.

(a) Frequency = frequency density × class width
$2.8 \times 20 = 56$
(b) 15 to 20: $1 \times 5 = 5$
Class 20 to 30: $2.2 \times 10 = 22$
Class 30 to 50: $2.8 \times 20 = 56$
Total = 5 + 22 + 56 = 83
Answer: (a) 56 (b) 83

10 Quadratic inequalities

Solve $x^2 - 8x \geq 0$

Solve $x^2 - 8x = 0$: $x(x - 8) = 0$
x = 0 or x = 8
Sketch the U-shaped curve $y = x^2 - 8x$, crossing the x-axis at 0 and 8
The curve is above the x-axis outside the roots
$x \leq 0$ or $x \geq 8$
Answer: $x \leq 0$ or $x \geq 8$