Day 2 **A Tier 10-a-day: Model Solutions**

1 nth term of a linear sequence

Here are the first four terms of a sequence.
12, 21, 30, 39
(a) Find an expression for the nth term.
(b) Work out the 40th term.

The terms go up by 9 each time, so the nth term starts with 9n
12 − 9 = 3, so the nth term is 9n + 3
(b) 40 × 9 + 3 = 363
Answer: (a) 9n + 3 (b) 363

2 Angles in parallel lines

The diagram shows two parallel lines crossed by a straight line.
(a) Work out the value of x.
(b) Work out the size of the angle marked (5x + 5)°.
The diagram is not drawn accurately.

(a) Co-interior angles add up to 180°
5x + 5 + 3x + 47 = 180
8x + 52 = 180
8x = 128
x = 16
(b) 5 × 16 + 5 = 85
Answer: (a) x = 16 (b) 85°

3 HCF and LCM

(a) Write 110 and 30 as products of prime factors.
(b) Find the highest common factor (HCF) and the lowest common multiple (LCM) of 110 and 30.

(a) 110 = 2 × 5 × 11 and 30 = 2 × 3 × 5
(b) HCF: multiply the prime factors in both lists, taking the smaller power: 2 × 5 = 10
LCM: take the larger power of every prime: 2 × 3 × 5 × 11 = 330
Answer: (a) 110 = 2 × 5 × 11, 30 = 2 × 3 × 5 (b) HCF = 10, LCM = 330

4 Changing a ratio

A bag contains red counters and blue counters in the ratio 5 : 6.
8 more red counters are added to the bag.
The ratio of red counters to blue counters is now 11 : 10.
How many counters were in the bag at the start?

Let red = 5x and blue = 6x. After adding: (5x + 8) : 6x = 11 : 10
$\frac{5x + 8}{6x} = \frac{11}{10}$, so 10(5x + 8) = 11 × 6x
50x + 80 = 66x, so 16x = 80, x = 5
Start: 5x + 6x = 11 × 5 = 55 counters
Answer: 55

5 Error intervals

m = 38.6 has been truncated to 1 decimal place.
Write down the error interval for m.

Truncating means cutting off the digits after the first decimal place.
The original value is at least 38.6 and less than 38.6 + 0.1 = 38.7
Answer: 38.6 ≤ m < 38.7

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6 Exact trigonometric values

Without using a calculator, work out the exact value of
$4 \cos 60° - 4 \tan 30°$
Give your answer in its simplest form.

$\cos 60° = \frac{1}{2}$ and $\tan 30° = \frac{\sqrt{3}}{3}$

$4 \times \frac{1}{2} - 4 \times \frac{\sqrt{3}}{3} = \frac{6 - 4\sqrt{3}}{3}$

Answer: $\mathbf{\frac{6 - 4\sqrt{3}}{3}}$

7 Composite functions

$f(x) = 3x - 2$
$g(x) = x^2 - 1$
Solve $gf(x) = 80$

$gf(x) = (3x - 2)^2 - 1$
$(3x - 2)^2 - 1 = 80$, so $(3x - 2)^2 = 81$
$3x - 2 = \pm 9$
$3x - 2 = 9$ or $3x - 2 = -9$
$x = \frac{11}{3}$ or $x = -\frac{7}{3}$

Answer: $\mathbf{x = \frac{11}{3}}$ **or** $\mathbf{x = -\frac{7}{3}}$

8 Simple and compound interest

Isla and Nia each invest £5000 for 2 years. (Calculator)
Isla's account pays 6% simple interest per year.
Nia's account pays 6% compound interest per year.
Work out how much more interest Nia earns than Isla.

Simple interest = £5000 × 6% × 2 = £600
Compound: £5000 × 1.06^2 = £5618.00, so interest = £618.00
Difference = 618.00 − 600 = £18.00 (to the nearest penny)
Answer: £18.00

9 Quadratic inequalities

Solve $x^2 + 4x - 5 > 0$

Solve $x^2 + 4x - 5 = 0$: $(x + 5)(x - 1) = 0$
$x = -5$ or $x = 1$
Sketch the U-shaped curve $y = x^2 + 4x - 5$, crossing the x-axis at -5 and 1
The curve is above the x-axis outside the roots
$x < -5$ or $x > 1$
Answer: $\mathbf{x < -5}$ **or** $\mathbf{x > 1}$

10 Probability with three selections

A bag contains 5 red counters and 5 blue counters.
Three counters are taken at random without replacement.
Work out the probability that all three counters are red.

P(all red) $= \frac{5}{10} \times \frac{4}{9} \times \frac{3}{8}$

$= \frac{60}{720} = \frac{1}{12}$

Answer: $\mathbf{\frac{1}{12}}$