Day 199 **A Tier 10-a-day: Model Solutions**

1 Isosceles triangles and exterior angles

PQR is an isosceles triangle with PQ = PR.
QRS is a straight line.
Angle QPR = 32°.
Work out the size of angle x.
The diagram is not drawn accurately.

Base angles of an isosceles triangle are equal
Angle PQR = (180 − 32) ÷ 2 = 74°
Exterior angle of a triangle = sum of the two opposite interior angles
x = 32 + 74 = 106
Answer: x = 106°

2 Exchange rates

The exchange rate is £1 = €1.15. (Calculator)
A hotel room costs €391 a night.
Work out the cost in pounds.

Divide by the exchange rate: 391 ÷ 1.15
= £340
Answer: £340

3 Volume of a cylinder

A cylinder has radius 6 cm. It contains water to a depth of 17 cm. (Calculator)
All the water is poured into an empty tank with a rectangular base 25 cm by 8 cm.
Work out the depth of the water in the tank. Give your answer to 3 significant figures.

Volume of water = $\pi \times 6^2 \times 17 = 612\pi$ cm^3
Depth = volume ÷ base area = $612\pi \div (25 \times 8)$
= 9.61
Answer: 9.61 cm

4 Mean from a frequency table

The table shows the results from 50 hens.
(a) Work out the mean.
(b) Write down the mode.

(a) Total = 1 × 13 + 2 × 5 + 3 × 20 + 4 × 4 + 5 × 8 = 139
Mean = 139 ÷ 50 = 2.78
(b) The highest frequency is 20, so the mode is 3
Answer: (a) 2.78 (b) 3

5 Quadratic graphs

The curve C has equation $y = x^2 - 6x + 8$
(a) Find the coordinates of the points where C crosses the x-axis.
(b) Find the coordinates of the turning point of C.

(a) Set y = 0: $x^2 - 6x + 8 = 0$
$(x - 4)(x - 2) = 0$
x = 2 or x = 4, so the graph crosses the x-axis at (2, 0) and (4, 0)
(b) The turning point is halfway between the roots: x = (2 + 4) ÷ 2 = 3
y = −1, so the minimum point is (3, −1)
Answer: (a) (2, 0) and (4, 0) (b) (3, −1)

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6 Bounds

A rectangle has length 14 cm and width 7 cm.
Each measurement is correct to the nearest cm.
Work out the upper bound for the perimeter of the rectangle.

Upper bound of length = 14.5 cm, upper bound of width = 7.5 cm
Perimeter = 2 × (14.5 + 7.5) = 44 cm
Answer: 44 cm

7 Forming quadratic equations

A rectangle has width x cm and length (x + 4) cm.
The area of the rectangle is 60 cm^2.
(a) Show that $x^2 + 4x - 60 = 0$
(b) Work out the value of x.

(a) Area = length × width, so $x(x + 4) = 60$
$x^2 + 4x = 60$
$x^2 + 4x - 60 = 0$
(b) Factorise: $(x - 6)(x + 10) = 0$
$x = 6$ or $x = -10$
A length cannot be negative, so $x = 6$
Answer: (a) Shown (b) x = 6

8 Column vectors

A is the point (−3, 1) and B is the point (−6, 5).
(a) Write →AB as a column vector.
(b) Work out the length of AB.

(a) $\rightarrow AB = \rightarrow OB - \rightarrow OA = \begin{pmatrix} -6 \\ 5 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$

(b) Length $= \sqrt{(-3)^2 + (4)^2} = \sqrt{25} = 5$

Answer: (a) $\begin{pmatrix} \mathbf{-3} \\ \mathbf{4} \end{pmatrix}$ **(b) 5**

9 Pressure

A block exerts a force of 20 N on the floor.
The area in contact with the floor is 0.2 m^2.
Work out the pressure on the floor in N/m^2.

Pressure = force ÷ area
= 20 ÷ 0.2 = 100
Answer: 100 N/m^2

10 Difference of two squares

(a) Factorise $x^2 - 100$
(b) Use the same idea to work out $7^2 - 3^2$ without a calculator.

(a) Difference of two squares: $x^2 - 100 = x^2 - 10^2$
$(x - 10)(x + 10)$
(b) $a^2 - b^2 = (a + b)(a - b)$, so $7^2 - 3^2 = (7 + 3)(7 - 3)$
$= 10 \times 4 = 40$
Answer: (a) (x − 10)(x + 10) (b) 40