Day 198 **A Tier 10-a-day: Model Solutions**

1 Standard form calculations

Work out $(4.5 \times 10^9) \times (5 \times 10^6)$
Give your answer in standard form.

Multiply the numbers and add the powers of 10: $4.5 \times 5 = 22.5$ and $10^9 \times 10^6 = 10^{15}$
22.5×10^{15}
22.5 is not between 1 and 10, so $= 2.25 \times 10^{16}$
Answer: 2.25×10^{16}

2 Converting speeds

A lorry travels at 27 km/h.
Work out this speed in metres per minute.

In 1 hour: 27 km = 27000 m
1 hour = 60 minutes, so $27000 \div 60 = 450$ m per minute
Answer: 450 m/min

3 Circle theorems: tangents

PA is a tangent to a circle with centre O. A is on the circle.
The radius is 15 cm and OP = 17 cm.
Work out the length of PA.
The diagram is not drawn accurately.

The tangent is perpendicular to the radius, so angle OAP = 90°
$PA^2 = 17^2 - 15^2 = 289 - 225 = 64$
$PA = \sqrt{64} = 8$
Answer: 8 cm

4 Quadratic graph from its roots

The diagram shows the graph of $y = x^2 + bx + c$.
Find the value of b and the value of c.

The curve crosses the x-axis at x = 3 and x = 4
So $y = (x - 3)(x - 4)$
Expand: $y = x^2 - 7x + 12$
$b = -7$ and $c = 12$
Answer: $b = -7$, $c = 12$

5 Successive percentage changes

The price of a games console is decreased by 40%.
The new price is then decreased by 30%.
Work out the overall percentage change.

First change: multiplier 0.6. Second change: multiplier 0.7
Overall multiplier $= 0.6 \times 0.7 = 0.42$
$0.42 - 1 = -0.58$, so the overall change is 58%
Answer: 58% decrease

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6 Parallel and perpendicular lines

The line L has equation $y = 5x + 1$

Find the equation of the line that is perpendicular to L and passes through the point (0, 3).

The gradient of L is 5

Perpendicular gradient = $-1 \div 5 = -\frac{1}{5}$

$y = -\frac{1}{5}x + c$, so $3 = -\frac{1}{5} \times 0 + c$

$c = 3$

$y = -\frac{1}{5}x + 3$

Answer: $y = -\frac{1}{5}x + 3$

7 Recurring decimals, show that

(a) Show that $0.\dot{7}\dot{5} = \frac{25}{33}$

(b) Hence write $5.\dot{7}\dot{5}$ as a mixed number in its simplest form.

(a) Let $x = 0.757575...$, so $100x = 75.7575...$

$100x - x = 75$, so $99x = 75$ and $x = \frac{75}{99} = \frac{25}{33}$

(b) $5.\dot{7}\dot{5} = 5 + \frac{25}{33} = 5\frac{25}{33}$

Answer: (b) $5\frac{25}{33}$

8 Exact trigonometric values

XYZ is a right-angled triangle with the right angle at Y.

XY = 8 cm and angle YXZ = 60°.

Work out the exact length of YZ.

Do not use a calculator.

YZ is opposite and XY is adjacent to the 60° angle, so use tan

$\tan 60° = \sqrt{3}$

$YZ = 8 \times \sqrt{3} = 8\sqrt{3}$

Answer: $8\sqrt{3}$ cm

9 Simultaneous equations (one quadratic)

Solve the simultaneous equations

$x^2 + y^2 = 25$

$y = 3x + 5$

Substitute $y = 3x + 5$ into $x^2 + y^2 = 25$

$x^2 + (3x + 5)^2 = 25$

$10x^2 + 30x = 0$

Divide by 10: $x^2 + 3x = 0$

$(x + 3)(x + 0) = 0$

$x = -3$ or $x = 0$

Use $y = 3x + 5$ to find y: when $x = -3$, $y = -4$; when $x = 0$, $y = 5$

Answer: $x = -3$, $y = -4$ and $x = 0$, $y = 5$

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10 Surface area of cones and spheres

A sphere has radius 15 cm.
Work out the surface area of the sphere.
Give your answer in terms of π.

Surface area = $4\pi r^2$
= $4 \times \pi \times 15^2$
= 900π

Answer: 900π cm^2