Day 193 **A Tier 10-a-day: Model Solutions**

1 Ratio and a difference

The ratio of cats to dogs in an animal shelter is 3 : 8.
There are 15 more dogs than cats.
Work out the number of cats.

Difference in parts = 8 − 3 = 5
5 parts = 15, so 1 part = 15 ÷ 5 = 3
cats: 3 × 3 = 9
Answer: 9

2 Area of parallelograms and kites

The area of this parallelogram is 55 cm^2.
Work out the value of h.
The diagram is not drawn accurately.

5 × h = 55
h = 55 ÷ 5 = 11
Answer: h = 11

3 Equation of a line from a graph

Work out the equation of the straight line.
Give your answer in the form y = mx + c

The line crosses the y-axis at (0, 1), so c = 1

Gradient = $\frac{\text{change in y}}{\text{change in x}}$: from (0, 1) to (−4, 7), y changes by 6 and x changes by −4

$m = -\frac{3}{2}$

$y = -\frac{3}{2}x + 1$

Answer: $\mathbf{y = -\frac{3}{2}x + 1}$

4 Fractions of an amount

Priya has £540.

Priya spends $\frac{1}{4}$ of the money on a bike and $\frac{1}{5}$ of the money that is left on a helmet.

How much money does Priya have left?

$\frac{1}{4}$ of £540 = £135, so after buying a bike, £405 is left.

$\frac{1}{5}$ of £405 = £81

£405 − £81 = £324
Answer: £324

5 Combined means

The mean mark of 13 boys in a test is 54.
The mean mark of 13 girls in the same test is 40.
Work out the mean mark of all 26 people.

Total for boys = 13 × 54 = 702
Total for girls = 13 × 40 = 520
Total = 1222, and number of students = 26
Mean = 1222 ÷ 26 = 47
Answer: 47

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6 Rearranging formulae (powers and roots)

Make x the subject of $y = 5\sqrt{x} + 1$

Subtract 1 from both sides: $y - 1 = 5\sqrt{x}$

Divide by 5: $\frac{y-1}{5} = \sqrt{x}$

Square both sides

Answer: $\mathbf{x = (\frac{y-1}{5})^2}$

7 Circle theorems: isosceles triangles with radii

A, B and C are points on a circle with centre O.
Angle OAB = 54°.
(a) Work out angle AOB.
(b) Work out the size of angle x.
Give reasons for your answers.

(a) OA = OB (radii), so triangle OAB is isosceles and angle OBA = 54°
Angle AOB = 180 − 54 − 54 = 72°
(b) The angle at the centre is twice the angle at the circumference
x = 72 ÷ 2 = 36
Answer: (a) 72° (b) x = 36°

8 Surd fractions

Show that $\frac{\sqrt{150} - \sqrt{96}}{\sqrt{6}}$ is an integer, and find its value.

$\sqrt{150} = 5\sqrt{6}$ and $\sqrt{96} = 4\sqrt{6}$
Numerator = $5\sqrt{6} - 4\sqrt{6} = \sqrt{6}$
$\frac{\sqrt{6}}{\sqrt{6}} = 1$
Answer: 1

9 Reverse compound change

The value of a painting increases by 2% each year. (Calculator)
After 2 years it is worth £41,616.
Work out its value at the start.

Yearly multiplier = 1.02
Overall multiplier = $1.02^2 = 1.0404$
Original value = £41,616 ÷ 1.0404 = £40,000
Answer: £40,000

10 Bearings and distances

A ship leaves a port and travels 12 miles on a bearing of 225°. (Calculator)
It then travels 5 miles on a bearing of 315°.
(a) Work out how far it is now from the port.
(b) Work out the bearing of its final position from the port. Give your answer to the nearest degree.

The two bearings differ by 90°, so the path makes a right angle
(a) $\text{Distance}^2 = 12^2 + 5^2 = 169$, so distance = 13 miles
(b) The angle at the port between the first leg and the line to the end: $\tan^{-1}(\frac{5}{12}) = 22.6°$
Bearing = 225° + 22.6° = 247.6°
Bearing = 248°
Answer: (a) 13 miles (b) 248°