Day 19 **A Tier 10-a-day: Model Solutions**

1 Loci and constructions

A radio mast is at point A. Its signal reaches up to 9 km.
A radio mast at B reaches up to 11 km.
Describe the region where both signals can be received.

Each signal covers the inside of a circle
Both signals are received where the two circles overlap
Answer: Inside both circles: the overlap of a circle of radius 9 km centred on A and a circle of radius 11 km centred on B

2 Prime factors

Express 2420 as a product of prime factors. Give your answer in index form.

Divide by prime numbers in turn: 2420 ÷ 2 = 1210, 1210 ÷ 2 = 605, 605 ÷ 5 = 121, 121 ÷ 11 = 11, 11 ÷ 11 = 1
$2420 = 2 \times 2 \times 5 \times 11 \times 11$
In index form: $2420 = 2^2 \times 5 \times 11^2$
Answer: $2420 = 2^2 \times 5 \times 11^2$

3 Surface area of a cuboid

A closed gift box is a cuboid measuring 6 cm by 6 cm by 2 cm.
Work out the total surface area.

Faces: 6 × 6, 6 × 2 and 6 × 2 each appear twice
$2 \times (36 + 12 + 12)$
$= 2 \times 60 = 120$
Answer: 120 cm^2

4 Expanding and simplifying

Expand and simplify
$3(x - 1)(x - 1) - (x - 1)(x + 1)$

$3(x - 1)(x - 1) = 3(x^2 - 2x + 1) = 3x^2 - 6x + 3$
$(x - 1)(x + 1) = x^2 - 1$
Subtract: $2x^2 - 6x + 4$
Answer: $2x^2 - 6x + 4$

5 Trigonometry: finding an angle

Work out the size of angle x. (Calculator)
Give your answer to 1 decimal place.
The diagram is not drawn accurately.

The sides given are the adjacent (9) and the hypotenuse (13), so use cos
$\cos x = \frac{9}{13}$
$x = \cos^{-1}(\frac{9}{13})$
$x = 46.2°$
Answer: x = 46.2°

6 Simple and compound interest

Grace and Femi each invest £2500 for 3 years. (Calculator)
Grace's account pays 3% simple interest per year.
Femi's account pays 3% compound interest per year.
Work out how much more interest Femi earns than Grace.

Simple interest = £2500 × 3% × 3 = £225
Compound: £2500 × 1.03^3 = £2731.818, so interest = £231.818
Difference = 231.818 − 225 = £6.82 (to the nearest penny)
Answer: £6.82

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7 Rationalising the denominator

Rationalise the denominator of $\frac{14}{2\sqrt{7}}$

Give your answer in its simplest form.

Multiply top and bottom by $\sqrt{7}$: $\frac{14\sqrt{7}}{2 \times 7}$

$= \frac{14\sqrt{7}}{14} = \sqrt{7}$

Answer: $\sqrt{7}$

8 Quadratic graphs

The curve C has equation $y = -x^2 + 6x - 8$

(a) Find the coordinates of the points where C crosses the x-axis.

(b) Find the coordinates of the turning point of C.

(a) Set $y = 0$: $-x^2 + 6x - 8 = 0$, so $x^2 - 6x + 8 = 0$ (divide by -1)

$(x - 4)(x - 2) = 0$

$x = 2$ or $x = 4$, so the graph crosses the x-axis at (2, 0) and (4, 0)

(b) The turning point is halfway between the roots: $x = (2 + 4) \div 2 = 3$

$y = 1$, so the maximum point is (3, 1)

Answer: (a) (2, 0) and (4, 0) (b) (3, 1)

9 Probability with three selections

A bag contains 3 red counters and 4 blue counters.

Three counters are taken at random without replacement.

Work out the probability that at least one of the three counters is blue.

P(none is blue) = P(all red) = $\frac{3}{7} \times \frac{2}{6} \times \frac{1}{5} = \frac{1}{35}$

P(at least one blue) = $1 - \frac{1}{35} = \frac{34}{35}$

Answer: $\frac{34}{35}$

10 Simultaneous equations (one quadratic)

Solve the simultaneous equations

$x^2 + y^2 = 25$

$y = 2x - 10$

Substitute $y = 2x - 10$ into $x^2 + y^2 = 25$

$x^2 + (2x - 10)^2 = 25$

$5x^2 - 40x + 75 = 0$

Divide by 5: $x^2 - 8x + 15 = 0$

$(x - 3)(x - 5) = 0$

$x = 3$ or $x = 5$

Use $y = 2x - 10$ to find y: when $x = 3$, $y = -4$; when $x = 5$, $y = 0$

Answer: $x = 3$, $y = -4$ and $x = 5$, $y = 0$