Day 188 **A Tier 10-a-day: Model Solutions**

1 Time

(a) A train leaves London at 2025. The journey takes 1 hours 22 minutes. Work out the time it ends.
(b) Change 2.2 hours into hours and minutes.

(a) 2025 + 1 hours = 2125, then + 22 minutes = 2147
(b) 0.2 hours = 0.2 × 60 = 12 minutes, so 2.2 hours = 2 hours 12 minutes
Answer: (a) 2147 (b) 2 hours 12 minutes

2 Quadratic graphs

The curve C has equation $y = x^2 - 4x - 21$
(a) Find the coordinates of the points where C crosses the x-axis.
(b) Find the coordinates of the turning point of C.

(a) Set y = 0: $x^2 - 4x - 21 = 0$
$(x - 7)(x + 3) = 0$
$x = -3$ or $x = 7$, so the graph crosses the x-axis at $(-3, 0)$ and $(7, 0)$
(b) The turning point is halfway between the roots: $x = (-3 + 7) \div 2 = 2$
$y = -25$, so the minimum point is $(2, -25)$
Answer: (a) $(-3, 0)$ and $(7, 0)$ (b) $(2, -25)$

3 Estimation

By rounding each number to 1 significant figure, estimate the value of

$$\frac{5.9 \times 23}{0.81}$$

Round each number to 1 significant figure: 6, 20, 0.8

$$\frac{6 \times 20}{0.8} = 150$$

Answer: 150

4 Combining ratios

a : b = 4 : 5
b : c = 2 : 5
(a) Work out the ratio a : b : c
(b) a, b and c add up to 86. Work out the value of c.

(a) Make the b parts equal: lowest common multiple of 5 and 2 is 10
a : b = 8 : 10 and b : c = 10 : 25
a : b : c = 8 : 10 : 25
(b) 8 + 10 + 25 = 43 parts, so 1 part = 86 ÷ 43 = 2
c = 25 × 2 = 50
Answer: (a) 8 : 10 : 25 (b) 50

5 Successive percentage changes

The price of a product increases by 12% each year for 2 years.
Work out the overall percentage increase over the 2 years.

Multiplier for each year = 1.12
Overall multiplier = $1.12^2 = 1.2544$
Percentage increase = 25.44%
Answer: 25.44%

6 Describing transformations

Describe fully the single transformation that maps shape A onto shape B.

Matching points are (x, y) and $(-y, -x)$
So the mirror line is the diagonal $y = -x$
Answer: Reflection in the line $y = -x$

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7 Quadratic graph from its roots

The diagram shows the graph of $y = -x^2 + bx + c$.
Find the value of b and the value of c.

The curve crosses the x-axis at $x = -5$ and $x = -2$
So $y = -(x + 5)(x + 2)$
Expand: $y = -x^2 - 7x - 10$
$b = -7$ and $c = -10$
Answer: $b = -7$, $c = -10$

8 Surface area of cones and spheres

A solid hemisphere has radius 11 cm.
Work out the total surface area of the hemisphere.
Give your answer in terms of π.

Curved surface = $\frac{1}{2} \times 4\pi r^2 = 242\pi$

Flat circular face = $\pi r^2 = 121\pi$
Total = 363π
Answer: 363π cm^2

9 Circles and tangents

The point P(3, 4) lies on a circle with centre O(0, 0).
(a) Work out the equation of the circle, in the form $x^2 + y^2 = r^2$.
(b) Find the equation of the tangent to the circle at P.

(a) The circle is $x^2 + y^2 = r^2$, so $r^2 = 3^2 + 4^2 = 25$

(b) Gradient of the radius OP = $\frac{4}{3}$

The tangent is perpendicular to the radius, so its gradient = $-1 \div \frac{4}{3} = -\frac{3}{4}$

$y = -\frac{3}{4}x + c$ through (3, 4): $4 = -\frac{3}{4} \times 3 + c$, so $c = \frac{25}{4}$

$y = -\frac{3}{4}x + \frac{25}{4}$

Answer: (a) $x^2 + y^2 = 25$ (b) $y = -\frac{3}{4}x + \frac{25}{4}$

10 Probability without replacement

A bag contains 3 mint and 6 fruit sweets.
Two sweets are taken at random without replacement.
Work out the probability that the two sweets are different colours.

P(same colour) = $\frac{3}{9} \times \frac{2}{8} + \frac{6}{9} \times \frac{5}{8} = \frac{1}{2}$

P(different) = $1 - \frac{1}{2} = \frac{1}{2}$

Answer: $\frac{1}{2}$