Day 186 **A Tier 10-a-day: Model Solutions**

1 Pythagoras in context

A ladder reaches 24 m up a vertical wall.
The ladder is 30 m long.
Work out how far the foot of the ladder is from the wall.

$d^2 + 24^2 = 30^2$
$d^2 = 900 - 576 = 324$
$d = 18$
Answer: 18 m

2 Mixed number addition and subtraction

Work out $5\frac{4}{5} - 3\frac{1}{2}$
Give your answer as a mixed number in its simplest form.

Write as improper fractions: $5\frac{4}{5} = \frac{29}{5}$ and $3\frac{1}{2} = \frac{7}{2}$
Common denominator 10: $\frac{58}{10} - \frac{35}{10}$
$= \frac{23}{10}$
$= 2\frac{3}{10}$
Answer: $2\frac{3}{10}$

3 Surface area of a cuboid

A cuboid has length 7 cm and width 11 cm.
Its total surface area is 298 cm^2.
Work out the height of the cuboid.

$2 \times (7 \times 11 + 7\,h + 11\,h) = 298$
$77 + 18h = 149$
$18h = 72$
$h = 72 \div 18 = 4$
Answer: 4 cm

4 Calculator and significant figures

Work out $6.3 \times (1 + 2.8 \div 100)^3$ (Calculator)
Give your answer correct to 3 significant figures.

$1 + 2.8 \div 100 = 1.028$
$= 6.84416...$
= 6.84 to 3 significant figures
Answer: 6.84

5 Direct and inverse proportion

y is directly proportional to the cube of x.
When x = 4, y = 192.
(a) Find a formula for y in terms of x.
(b) Work out the value of y when x = 6.

(a) $y = kx^3$. Substitute x = 4 and y = 192: $192 = k \times 64$, so $k = 3$
$y = 3x^3$
(b) $y = 3 \times 6^3 = 3 \times 216 = 648$
Answer: (a) $y = 3x^3$ (b) 648

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6 Combining ratios

p : q = 5 : 6
q : r = 6 : 1
(a) Work out the ratio p : q : r
(b) p, q and r add up to 36. Work out the value of r.

(a) Make the q parts equal: lowest common multiple of 6 and 6 is 6
p : q = 5 : 6 and q : r = 6 : 1
p : q : r = 5 : 6 : 1
(b) 5 + 6 + 1 = 12 parts, so 1 part = 36 ÷ 12 = 3
r = 1 × 3 = 3
Answer: (a) 5 : 6 : 1 (b) 3

7 The cosine rule: finding a side

Work out the value of x. (Calculator)
Give your answer to 3 significant figures.
The diagram is not drawn accurately.

$x^2 = 11^2 + 13^2 - 2 \times 11 \times 13 \times \cos 64°$
$x^2 = 121 + 169 - 286 \cos 64°$
$x^2 = 164.63$
$x = 12.8$
Answer: x = 12.8 cm

8 Frequency density

The table gives information about 166 measurements.
A histogram is drawn using this information.
(a) Work out the missing frequency.
(b) Work out the height of the bar for the class $0 < t \leq 5$.

(a) Missing frequency = 166 − (4 + 78 + 66) = 18
(b) Class width = 5 − 0 = 5
Frequency density = 4 ÷ 5 = 0.8
Answer: (a) 18 (b) 0.8

9 Fractions of an amount

Ben spends $\frac{3}{4}$ of the money on a book and $\frac{3}{5}$ of the money that is left on a game.
Ben has £36 left.
How much money did Ben have to begin with?

After the second purchase $\frac{2}{5}$ of what was left after the first purchase remains.
£36 = $\frac{2}{5}$ of the amount left, so $\frac{1}{5}$ of it is £36 ÷ 2 = £18
So the amount left after the first purchase is £18 × 5 = £90
This is $\frac{1}{4}$ of the original amount, so the original amount is £90 × 4 = £360
Answer: £360

10 Algebraic rearrangement and change of sign

$f(x) = x^3 - 3x^2 + 2x + 7$
Show that the equation f(x) = 0 has a solution between x = −2 and x = −1.

f(−2) = −17
f(−1) = 1
There is a change of sign and f is continuous, so there is a solution between x = −2 and x = −1
Answer: f(−2) = −17 and f(−1) = 1 (change of sign)