Day 185 **A Tier 10-a-day: Model Solutions**

1 Ordering numbers

Write these numbers in order of size, starting with the smallest.

73% $\frac{5}{8}$ 0.79 0.33

Change each to a decimal: 73% = 0.73, $\frac{5}{8}$ = 0.625, 0.79 = 0.79, 0.33 = 0.33

Order the decimals: 0.33 < 0.625 < 0.73 < 0.79

Answer: 0.33, $\frac{5}{8}$, 73%, 0.79

2 Equation of a straight line

A straight line passes through the points (1, 3) and (4, 9).
Find the equation of the line.

Gradient = $\frac{9-3}{4-1} = \frac{6}{3} = 2$

Use y = mx + c with m = 2 and the point (1, 3): 3 = 2 × 1 + c
c = 1
y = 2x + 1
Answer: y = 2x + 1

3 Combining ratios

a : b = 4 : 1
b : c = 5 : 7
(a) Work out the ratio a : b : c
(b) a, b and c add up to 64. Work out the value of c.

(a) Make the b parts equal: lowest common multiple of 1 and 5 is 5
a : b = 20 : 5 and b : c = 5 : 7
a : b : c = 20 : 5 : 7
(b) 20 + 5 + 7 = 32 parts, so 1 part = 64 ÷ 32 = 2
c = 7 × 2 = 14
Answer: (a) 20 : 5 : 7 (b) 14

4 Circle theorems: angle in a semicircle

PQ is a diameter of the circle.
Angle QPR = 53°.
Work out the size of angle x.
Give a reason for each step of your working.

The angle in a semicircle is 90°, so angle PRQ = 90°
Angles in a triangle add up to 180°
x = 180 − 90 − 53 = 37
Answer: x = 37°

5 Rates of flow

A tank is in the shape of a cuboid measuring 50 cm by 40 cm by 100 cm.
It already contains 20 litres of water.
Water flows in at 12 litres per minute until the tank is full.
Work out how long this takes. Give your answer in minutes.

Volume = 50 × 40 × 100 = 200000 cm^3 = 200 litres
Water needed = 200 − 20 = 180 litres
Time = 180 ÷ 12 = 15 minutes
Answer: 15 minutes

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6 Rationalising the denominator

Rationalise the denominator of $\frac{4}{4\sqrt{2}}$

Give your answer in its simplest form.

Multiply top and bottom by $\sqrt{2}$: $\frac{4\sqrt{2}}{4 \times 2}$

$= \frac{4\sqrt{2}}{8} = \frac{\sqrt{2}}{2}$

Answer: $\frac{\sqrt{2}}{2}$

7 Algebraic rearrangement and change of sign

(a) Show that the equation $x^3 + 5x - 7 = 0$ can be rearranged to give $x = \sqrt[3]{7 - 5x}$

(b) Show that the equation has a solution between $x = 1$ and $x = 2$

(a) Add 7 and subtract 5x from both sides: $x^3 = 7 - 5x$

Take the cube root of both sides: $x = \sqrt[3]{7 - 5x}$

(b) Let $f(x) = x^3 + 5x - 7$

$f(1) = -1$

$f(2) = 11$

There is a change of sign (negative to positive) and f is continuous, so there is a solution between $x = 1$ and $x = 2$

Answer: Shown: $f(1) = -1$, $f(2) = 11$ (change of sign)

8 Changing a ratio

A bag contains red counters and blue counters in the ratio 4 : 1.

7 more red counters are added to the bag.

The ratio of red counters to blue counters is now 9 : 2.

How many counters were in the bag at the start?

Let red = 4x and blue = 1x. After adding: (4x + 7) : 1x = 9 : 2

$\frac{4x + 7}{1x} = \frac{9}{2}$, so $2(4x + 7) = 9 \times 1x$

$8x + 14 = 9x$, so $1x = 14$, $x = 14$

Start: $4x + 1x = 5 \times 14 = 70$ counters

Answer: 70

9 Simultaneous equations in context

2 footballs and 2 netballs cost £26.

4 footballs and 2 netballs cost £36.

Work out the cost of one netball.

Let f = cost of one football in £ and n = cost of one netball in £

$2f + 2n = 26$ (1)

$4f + 2n = 36$ (2)

The n terms have the same sign, so subtract equation (1) from equation (2)

$2f = 10$, so $f = 5$

Substitute $f = 5$ into equation (1) to find $n = 8$

Answer: £8

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10 Circle theorems: angles in the same segment

A, B, C and D are points on a circle. AC and BD meet at E.
Angle BAC = 57° and angle ACD = 48°.
Work out the size of angle x (angle AEB).
Give reasons for your working.

Angles in the same segment are equal, so angle ABD = angle ACD = 48°
In triangle ABE: angle BAE = 57° and angle ABE = 48°
x = 180 − 57 − 48 = 75
Answer: x = 75°