Day 184 **A Tier 10-a-day: Model Solutions**

1 Primes

(a) Write down all the prime numbers between 50 and 70.
(b) Write 119 as the product of two prime numbers.

(a) A prime number has exactly two factors. Checking each number between 50 and 70 gives: 53, 59, 61, 67
(b) 119 ÷ 7 = 17, and both 7 and 17 are prime.
Answer: (a) 53, 59, 61, 67 (b) 7 × 17

2 Isosceles triangles and exterior angles

ABC is an isosceles triangle with AB = AC.
BCD is a straight line.
Angle BAC = 80°.
Work out the size of angle x.
The diagram is not drawn accurately.

Base angles of an isosceles triangle are equal
Angle ABC = (180 − 80) ÷ 2 = 50°
Exterior angle of a triangle = sum of the two opposite interior angles
x = 80 + 50 = 130
Answer: x = 130°

3 HCF and LCM

(a) Write 50 and 55 as products of prime factors.
(b) Find the highest common factor (HCF) and the lowest common multiple (LCM) of 50 and 55.

(a) $50 = 2 \times 5^2$ and $55 = 5 \times 11$
(b) HCF: multiply the prime factors in both lists, taking the smaller power: 5
LCM: take the larger power of every prime: $2 \times 5^2 \times 11 = 550$
Answer: (a) $50 = 2 \times 5^2$, $55 = 5 \times 11$ (b) HCF = 5, LCM = 550

4 Probability without replacement

A bag contains 2 red and 7 blue counters.
Two counters are taken at random without replacement.
Work out the probability that the two counters are different colours.

$$\text{P(same colour)} = \frac{2}{9} \times \frac{1}{8} + \frac{7}{9} \times \frac{6}{8} = \frac{11}{18}$$

$$\text{P(different)} = 1 - \frac{11}{18} = \frac{7}{18}$$

Answer: $\frac{7}{18}$

5 Quadratic graphs

The curve C has equation $y = -x^2 + 25$
(a) Find the coordinates of the points where C crosses the x-axis.
(b) Find the coordinates of the turning point of C.

(a) Set y = 0: $-x^2 + 25 = 0$, so $x^2 - 25 = 0$ (divide by −1)
$(x - 5)(x + 5) = 0$
x = −5 or x = 5, so the graph crosses the x-axis at (−5, 0) and (5, 0)
(b) The turning point is halfway between the roots: x = (−5 + 5) ÷ 2 = 0
y = 25, so the maximum point is (0, 25)
Answer: (a) (−5, 0) and (5, 0) (b) (0, 25)

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6 Successive percentage changes

The price of a ticket is increased by 20% and then the new amount is decreased by 40%.
Work out the overall percentage decrease.

Multipliers: increase of 20% is 1.2, decrease of 40% is 0.6
Overall multiplier = 1.2 × 0.6 = 0.72
0.72 means a percentage decrease of 28%
Answer: 28% decrease

7 Adding algebraic fractions

Write as a single fraction in its simplest form

$\frac{x+1}{3} + \frac{x-6}{4}$

Common denominator is 12

$= \frac{4(x+1) + 3(x-6)}{12}$

$= \frac{7x-14}{12}$

Answer: $\frac{7x-14}{12}$

8 Trigonometry in context

A tree stands on horizontal ground. (Calculator)
From a point 85 m from the base of the tree, the angle of elevation of the top is 29°.
Work out the height of the tree. Give your answer to 3 significant figures.

$\tan 29° = \frac{\text{height}}{85}$

height = 85 × tan 29°
= 47.1
Answer: 47.1 m

9 Mixing in a ratio

Jug A contains 36 litres of a mixture of concentrate and water in the ratio 7 : 5.
Jug B contains 20 litres of a mixture of concentrate and water in the ratio 3 : 7.
The contents of both jugs are mixed together.
Work out the ratio of concentrate to water in the mixture. Give your answer in its simplest form.

Jug A: concentrate = 7 × 36 ÷ 12 = 21 litres, water = 15 litres
Jug B: concentrate = 6 litres, water = 14 litres
Total: concentrate = 27 litres, water = 29 litres
Ratio = 27 : 29 = 27 : 29
Answer: 27 : 29

10 The cosine rule: finding an angle

Work out the size of angle x. (Calculator)
Give your answer to 1 decimal place.
The diagram is not drawn accurately.

$\cos x = \frac{11^2 + 15^2 - 14^2}{2 \times 11 \times 15}$

$\cos x = \frac{150}{330}$

$x = \cos^{-1}(0.4545)$
$x = 63.0°$
Answer: x = 63.0°