Day 180 **A Tier 10-a-day: Model Solutions**

1 Converting units

(a) Change 1.2 cm^2 to mm^2.
(b) Change 450 mm^2 to cm^2.

(a) 1.2 cm^2 × 100 = 120 mm^2
(b) 450 mm^2 ÷ 100 = 4.5 cm^2
Answer: (a) 120 mm^2 (b) 4.5 cm^2

2 Forming and solving equations

The sum of 4 consecutive whole numbers is 22.
Work out the largest of the numbers.

Let the smallest number be n. The numbers are n, n + 1, n + 2, n + 3
Sum = 4n + 6 = 22
4n = 16
n = 4
The numbers are 4, 5, 6, 7
Answer: 7

3 Reverse percentages

In a sale, all prices are reduced by 25%.
A tent costs £60 in the sale.
Work out the original price of the tent.

75% represents £60
1% = £60 ÷ 75 = £0.8
100% = £0.8 × 100 = £80.00
Answer: £80.00

4 Volume of a hollow cylinder

The diagram shows the cross-section of a metal pipe.
The outer radius is 4 cm and the inner radius is 3 cm.
The pipe is 12 cm long.
Work out the volume of metal used. Give your answer to 3 significant figures. (Calculator)

Area of the ring = $\pi \times 4^2 - \pi \times 3^2 = \pi(16 - 9) = 7\pi$
Volume = area × length = $7\pi \times 12$
= 264
Answer: 264 cm^3

5 Finding a missing value from an average

The mean of the five numbers 4, 23, x, 10, 11 is 13.
Work out the value of x.

Total = 5 × 13 = 65
Known numbers add to 48
x = 65 − 48 = 17
Answer: x = 17

6 Expanding with surds

Expand and simplify $(3 + \sqrt{2})^2$
Give your answer in the form $a + b\sqrt{c}$

$(3 + \sqrt{2})(3 + \sqrt{2}) = 9 + 3\sqrt{2} + 3\sqrt{2} + 2$
Collect: $11 + 6\sqrt{2}$
Answer: $11 + 6\sqrt{2}$

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7 Loci: area of a region

A goat is tied by a rope of length 6 m to the corner of a square field of side 10 m.
The goat can graze inside the field only.
Work out the area it can graze. Give your answer in terms of π.
The diagram is not drawn accurately.

The rope can sweep a quarter circle (90°)

Area = $\frac{1}{4} \times \pi \times 6^2$

= 9π

Answer: 9π m^2

8 Perpendicular bisector

A is the point (0, 1) and B is the point (−6, 3).
Find the equation of the perpendicular bisector of AB.

Midpoint of AB = $(\frac{0 + (-6)}{2}, \frac{1 + 3}{2}) = (-3, 2)$

Gradient of AB = $\frac{2}{-6} = -\frac{1}{3}$

Perpendicular gradient = $-1 \div (-\frac{1}{3}) = 3$

$y = 3x + c$ through $(-3, 2)$: $2 = 3 \times (-3) + c$, so $c = 11$
$y = 3x + 11$
Answer: $y = 3x + 11$

9 Adding algebraic fractions

Write as a single fraction in its simplest form

$\frac{x + 2}{6} - \frac{x - 4}{5}$

Common denominator is 30

= $\frac{5(x + 2) - 6(x - 4)}{30}$

= $\frac{-x + 34}{30}$

Answer: $\frac{-x + 34}{30}$

10 Pythagoras in three dimensions

The diagram shows a cuboid.
Work out the length of the diagonal AG.
The diagram is not drawn accurately.

$AG^2 = 4^2 + 3^2 + 12^2$
$= 16 + 9 + 144 = 169$
$AG = \sqrt{169} = 13$
Answer: 13 cm