Day 178 **A Tier 10-a-day: Model Solutions**

1 Circle theorems: tangents

PA and PB are tangents to a circle with centre O.
Angle APB = 98°.
Work out the size of angle x (angle OAB).
Give a reason for each step of your working.

A tangent is perpendicular to the radius, so angles OAP and OBP are both 90°
Angle AOB = 360 − 90 − 90 − 98 = 82°
OA = OB (radii), so triangle OAB is isosceles
x = (180 − 82) ÷ 2 = 49
Answer: x = 49°

2 HCF and LCM

(a) Write 44 and 28 as products of prime factors.
(b) Find the highest common factor (HCF) and the lowest common multiple (LCM) of 44 and 28.

(a) $44 = 2^2 \times 11$ and $28 = 2^2 \times 7$
(b) HCF: multiply the prime factors in both lists, taking the smaller power: $2^2 = 4$
LCM: take the larger power of every prime: $2^2 \times 7 \times 11 = 308$
Answer: (a) $44 = 2^2 \times 11$, $28 = 2^2 \times 7$ (b) HCF = 4, LCM = 308

3 Circle theorems: angle in a semicircle

AB is a diameter of the circle.
Angle BAC = 22°.
Work out the size of angle x.
Give a reason for each step of your working.

The angle in a semicircle is 90°, so angle ACB = 90°
Angles in a triangle add up to 180°
x = 180 − 90 − 22 = 68
Answer: x = 68°

4 Standard form notation

(a) Write 0.00037 in standard form.
(b) Write 5.07×10^4 as an ordinary number.

(a) Move the decimal point 4 places to get 3.7: 3.7×10^{-4}
(b) Move the decimal point 4 places to the right: 50,700
Answer: (a) 3.7×10^{-4} (b) 50,700

5 Equation of a straight line

A straight line passes through the points (−2, −1) and (−1, 2).
Find the equation of the line.

Gradient = $\frac{2-(-1)}{-1-(-2)} = \frac{3}{1}$
Use y = mx + c with m = 3 and the point (−2, −1): $-1 = 3 \times (-2) + c$
c = 5
y = 3x + 5
Answer: y = 3x + 5

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6 Two-way tables

The table shows information about 98 people.
One person is chosen at random.
(a) Find the probability that the person is a woman and chose pasta.
(b) The person chose chicken. Find the probability that they are a man.

(a) Women who chose pasta: 16 out of 98 = $\frac{8}{49}$

(b) Chicken total = 53, of whom 30 are in the Men row: $\frac{30}{53}$

Answer: (a) $\frac{8}{49}$ (b) $\frac{30}{53}$

7 Bounds of a speed or density

A metal block has mass 820 g and volume 72 cm^3. The mass is measured to the nearest 10 grams and the volume to the nearest cm^3. (Calculator)
Work out the upper bound for the density of the metal in g/cm^3.
Give your answer correct to 3 significant figures.

density = mass ÷ volume
For the upper bound of density use the upper bound of mass and the lower bound of volume.
825 ÷ 71.5 = 11.53846
= 11.5 g/cm^3 (3 s.f.)
Answer: 11.5 g/cm^3

8 Venn diagrams from information

There are 18 students in a class.
8 students do chess and 7 students do coding.
6 students do neither.
(a) Work out how many students do both.
(b) One student is chosen at random. Find the probability that they do chess only.

(a) Students doing at least one = 18 − 6 = 12
Both = 8 + 7 − 12 = 3
(b) chess only = 8 − 3 = 5

Probability = $\frac{5}{18}$

Answer: (a) 3 (b) $\frac{5}{18}$

9 Mixing in a ratio

Jug A contains 15 litres of a mixture of orange juice and lemonade in the ratio 2 : 1.
Jug B contains 8 litres of a mixture of orange juice and lemonade in the ratio 3 : 1.
The contents of both jugs are mixed together.
Work out the ratio of orange juice to lemonade in the mixture. Give your answer in its simplest form.

Jug A: orange juice = 2 × 15 ÷ 3 = 10 litres, lemonade = 5 litres
Jug B: orange juice = 6 litres, lemonade = 2 litres
Total: orange juice = 16 litres, lemonade = 7 litres
Ratio = 16 : 7 = 16 : 7
Answer: 16 : 7

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10 Laws of indices

Simplify fully

(a) $\frac{9x^5y^4}{2x^2y^6}$

(b) $(2x^3y)^3$

(a) Divide the numbers: $9 \div 2 = \frac{9}{2}$

Subtract the powers: $x^5 \div x^2 = x^3$ and $y^4 \div y^6 = y^{-2}$

$= \frac{9x^3}{2y^2}$

(b) Raise every part to the power 3: $2^3 = 8$, $(x^3)^3 = x^9$, $y^3 = y^3$

$= 8x^9y^3$

Answer: (a) $\frac{9x^3}{2y^2}$ (b) $8x^9y^3$