Day 177 **A Tier 10-a-day: Model Solutions**

1 Substitution

Work out the value of $5x^2 - 8x + 1$ when $x = -6$

Substitute $x = -6$: $5 \times (-6)^2 - 8 \times (-6) + 1$
$= 180 + 48 + 1$
$= 229$
Answer: 229

2 Pythagoras in context

A rectangular field measures 7 m by 24 m.
Femi walks along the diagonal of the field.
How far does Femi walk?

$\text{Diagonal}^2 = 7^2 + 24^2$
$= 49 + 576 = 625$
$\text{Diagonal} = \sqrt{625} = 25$
Answer: 25 m

3 Mean from a frequency table

The table shows the results from 25 students.
(a) Work out the mean.
(b) Write down the mode.

(a) Total $= 0 \times 6 + 1 \times 6 + 2 \times 10 + 3 \times 1 + 4 \times 2 = 37$
Mean $= 37 \div 25 = 1.48$
(b) The highest frequency is 10, so the mode is 2
Answer: (a) 1.48 (b) 2

4 Congruent triangles

Triangle DEF: DE = 10 cm, EF = 12 cm, DF = 11 cm.
Triangle PQR: PQ = 10 cm, QR = 12 cm, PR = 12 cm.
Are the two triangles congruent?
Give a reason for your answer.

The sides of triangle DEF are 10, 12 and 11
The sides of triangle PQR are 10, 12 and 12
The side lengths are not the same, so the triangles are not congruent
Answer: No, not congruent

5 Independent events

The probability that Owen scores a goal is 0.2.
The probability that Lena scores a goal is 0.9.
The two events are independent.
Work out the probability that neither Owen nor Lena will score a goal.

P(Owen does not) $= 1 - 0.2 = 0.8$ and P(Lena does not) $= 1 - 0.9 = 0.1$
$0.8 \times 0.1 = 0.08$
Answer: 0.08

6 Terminating and recurring decimals

Here are four fractions: $\frac{5}{16}$ $\frac{4}{5}$ $\frac{13}{40}$ $\frac{10}{24}$

Which one of these fractions is equal to a recurring decimal?

Simplify each fraction: $\frac{5}{16}, \frac{4}{5}, \frac{13}{40}, \frac{10}{24} = \frac{5}{12}$

A fraction in simplest form is a terminating decimal only if its denominator has no prime factors other than 2 and 5.

$\frac{5}{12}$ has denominator $12 = 2 \times 2 \times 3$, which includes a factor other than 2 and 5.

Answer: $\frac{10}{24}$

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7 Composite functions

f(x) = 3x − 3

g(x) = x^2 − 3

Solve gf(x) = 22

gf(x) = $(3x - 3)^2$ − 3

$(3x - 3)^2$ − 3 = 22, so $(3x - 3)^2$ = 25

3x − 3 = ±5

3x − 3 = 5 or 3x − 3 = −5

$x = \frac{8}{3}$ or $x = -\frac{2}{3}$

Answer: $x = \frac{8}{3}$ or $x = -\frac{2}{3}$

8 Bounds

A triangle has base 4.2 m and perpendicular height 4.6 m. (Calculator)

Each measurement is correct to the nearest 0.1 m.

Work out the upper bound for the area of the triangle.

Upper bounds: base = 4.25 m, height = 4.65 m

Area = ½ × 4.25 × 4.65 = 9.88125 m^2

Answer: 9.88125 m^2

9 Equations with algebraic fractions

Solve $\frac{6}{(x + 5)} + \frac{4}{(x - 2)} = 1$

Multiply every term by (x + 5)(x − 2)

6(x − 2) + 4(x + 5) = (x + 5)(x − 2)

10x + 8 = x^2 + 3x − 10

x^2 − 7x − 18 = 0

(x + 2)(x − 9) = 0

x = −2 or x = 9

Answer: x = −2 or x = 9

10 Bearings with the cosine rule

Boat P sails 17 miles from a harbour on a bearing of 250°.

Boat Q sails 13 miles from the same harbour on a bearing of 145°. (Calculator)

Work out the distance between the two boats.

Give your answer to 3 significant figures.

The angle between the two paths = |250° − 145°| = 105°

$d^2 = 17^2 + 13^2 - 2 \times 17 \times 13 \times \cos 105°$

d^2 = 572.40

d = 23.9

Answer: 23.9 miles