Day 173 **A Tier 10-a-day: Model Solutions**

1 Circle theorems: tangents

PA and PB are tangents to a circle with centre O.
Angle APB = 58°.
Work out the size of angle x (angle AOB).
Give a reason for each step of your working.

A tangent is perpendicular to the radius, so angles OAP and OBP are both 90°
The angles in quadrilateral OAPB add up to 360°
x = 360 − 90 − 90 − 58 = 122
Answer: x = 122°

2 Algebraic formulae in context

A gym charges a joining fee of £20 and then £30 each month.
(a) Write a formula for the total cost, £C, after n months.
(b) Maria pays £290 in total. Work out the number of months.

(a) C = 20 + 30n
(b) 20 + 30n = 290
30n = 270
n = 9
Answer: (a) C = 20 + 30n (b) 9

3 Calculator and significant figures

Work out $\frac{26.4 + 0.8}{7.5 - 0.14}$ (Calculator)
Give your answer correct to 3 significant figures.

Numerator = 26.4 + 0.8 = 27.2
Denominator = 7.5 − 0.14 = 7.36
= 3.69565...
= 3.70 to 3 significant figures
Answer: 3.70

4 Mean from a frequency table

The table shows information about 25 matches.
(a) Work out the mean.
(b) Write down the mode.

(a) Total = 0 × 7 + 1 × 6 + 2 × 4 + 3 × 2 + 4 × 6 = 44
Mean = 44 ÷ 25 = 1.76
(b) The highest frequency is 7, so the mode is 0
Answer: (a) 1.76 (b) 0

5 Midpoints and length of a line segment

A is the point (−3, 1) and B is the point (−4, −6).
(a) Find the midpoint of AB.
(b) Work out the length of AB. Give your answer as a surd in its simplest form.

(a) Midpoint = $(\frac{-3 + (-4)}{2}, \frac{1 + (-6)}{2})$ = (−3.5, −2.5)

(b) Length = $\sqrt{(-1)^2 + (-7)^2}$
= $\sqrt{50}$ = $5\sqrt{2}$
Answer: (a) (−3.5, −2.5) (b) $5\sqrt{2}$

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6 Completing the square

(a) Write $x^2 + 14x - 3$ in the form $(x + a)^2 + b$

(b) Write down the coordinates of the turning point of the curve $y = x^2 + 14x - 3$

(a) Half of 14 is 7, so $(x + 7)^2$

$(x + 7)^2 = x^2 + 14x + 49$, so subtract 49 to get back to -3

So $x^2 + 14x - 3 = (x + 7)^2 - 52$

(b) The turning point is where the bracket is 0, so $x = -7$ and $y = -52$

Answer: (a) $(x + 7)^2 - 52$ (b) $(-7, -52)$

7 Rationalising the denominator

Rationalise the denominator of $\frac{10}{\sqrt{27}}$

Give your answer in its simplest form.

Simplify the denominator: $\sqrt{27} = 3\sqrt{3}$

$\frac{10}{\sqrt{27}} = \frac{10}{3\sqrt{3}}$

Multiply top and bottom by $\sqrt{3}$: $\frac{10\sqrt{3}}{3 \times 3}$

$= \frac{10\sqrt{3}}{9}$

Answer: $\frac{10\sqrt{3}}{9}$

8 Inverse proportion formulae

y is inversely proportional to $\sqrt{x}$.

When $x = 1$, $y = 3$.

(a) Find a formula for y in terms of x.

(b) Work out the value of y when $x = 16$.

(a) $y = \frac{k}{\sqrt{x}}$. Substitute: $3 = \frac{k}{1}$, so $k = 3 \times 1 = 3$

$y = \frac{3}{\sqrt{x}}$

(b) $y = \frac{3}{4} = 0.75$

Answer: (a) $y = \frac{3}{\sqrt{x}}$ (b) 0.75

9 Expanding three brackets

Expand and simplify

$(x + 4)(x - 5)(x - 3)$

Expand the first two brackets: $(x + 4)(x - 5) = x^2 - x - 20$

Multiply by the third bracket: $(x^2 - x - 20)(x - 3)$

$= x^3 - x^2 - 20x - 3x^2 + 3x + 60$

$= x^3 - 4x^2 - 17x + 60$

Answer: $x^3 - 4x^2 - 17x + 60$

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10 Finding an interest rate

Jack invests £4000 in a savings account. (Calculator)
The account pays compound interest at a fixed rate each year.
After 2 years there is £5017.60 in the account.
Work out the annual rate of interest.

Let the yearly multiplier be m. Then $4000 \times m^2 = 5017.6$

$m^2 = 5017.6 \div 4000 = 1.2544$

$m = \sqrt{1.2544} = 1.12$

Rate = $(1.12 - 1) \times 100 = 12\%$

Answer: 12%