Day 171 **A Tier 10-a-day: Model Solutions**

1 Substitution

$s = ut + \frac{1}{2}at^2$

Work out the value of s when $u = -4$, $a = 6$ and $t = 6$

$s = (-4) \times 6 + \frac{1}{2} \times (6) \times 6^2$

$= -24 + (108)$

$= 84$

Answer: 84

2 Circle theorems: tangents

PA and PB are tangents to a circle with centre O.
Angle APB = 100°.
Work out the size of angle x (angle OAB).
Give a reason for each step of your working.

A tangent is perpendicular to the radius, so angles OAP and OBP are both 90°
Angle AOB = 360 − 90 − 90 − 100 = 80°
OA = OB (radii), so triangle OAB is isosceles
x = (180 − 80) ÷ 2 = 50
Answer: x = 50°

3 Frequency density

The table gives information about 180 measurements.
A histogram is drawn using this information.
(a) Work out the missing frequency.
(b) Work out the height of the bar for the class $40 < t \leq 70$.

(a) Missing frequency = 180 − (16 + 72 + 44) = 48
(b) Class width = 70 − 40 = 30
Frequency density = 72 ÷ 30 = 2.4
Answer: (a) 48 (b) 2.4

4 Error intervals

x = 0.69 is rounded to 2 significant figures.
Write down the error interval for x.

The rounding unit is 0.01, so the value could be up to half of this, 0.005, either side.
Lower bound = 0.69 − 0.005 = 0.685, upper bound = 0.69 + 0.005 = 0.695
Answer: $0.685 \leq x < 0.695$

5 Midpoints and length of a line segment

C is the point (−4, −4) and D is the point (−9, −16).
(a) Find the midpoint of CD.
(b) Work out the length of CD. Give your answer as a whole number.

(a) Midpoint = $(\frac{-4 + (-9)}{2}, \frac{-4 + (-16)}{2}) = (-6.5, -10)$

(b) Length = $\sqrt{(-5)^2 + (-12)^2}$

$= \sqrt{169} = 13$

Answer: (a) (−6.5, −10) (b) 13

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6 Rationalising the denominator

Rationalise the denominator of $\frac{14}{\sqrt{20}}$

Give your answer in its simplest form.

Simplify the denominator: $\sqrt{20} = 2\sqrt{5}$

$\frac{14}{\sqrt{20}} = \frac{14}{2\sqrt{5}}$

Multiply top and bottom by $\sqrt{5}$: $\frac{14\sqrt{5}}{2 \times 5}$

$= \frac{7\sqrt{5}}{5}$

Answer: $\frac{7\sqrt{5}}{5}$

7 Algebraic proof

Prove that $(n + 3)^2 - (n - 3)^2$ is a multiple of 4 for any integer n.

$(n + 3)^2 = n^2 + 6n + 9$

$(n - 3)^2 = n^2 - 6n + 9$

Difference = $12n = 4(3n)$

This is 4 × an integer, so it is a multiple of 4

Answer: Proved

8 Changing a ratio

A bag contains red counters and blue counters in the ratio 1 : 6.
15 more red counters are added to the bag.
The ratio of red counters to blue counters is now 3 : 8.
How many counters were in the bag at the start?

Let red = 1x and blue = 6x. After adding: (1x + 15) : 6x = 3 : 8

$\frac{1x + 15}{6x} = \frac{3}{8}$, so $8(1x + 15) = 3 \times 6x$

$8x + 120 = 18x$, so $10x = 120$, $x = 12$

Start: $1x + 6x = 7 \times 12 = 84$ counters

Answer: 84

9 Quadratic inequalities

Solve $x^2 - 14x + 40 < 0$

Solve $x^2 - 14x + 40 = 0$: $(x - 4)(x - 10) = 0$

$x = 4$ or $x = 10$

Sketch the U-shaped curve $y = x^2 - 14x + 40$, crossing the x-axis at 4 and 10

The curve is below the x-axis between the roots

$4 < x < 10$

Answer: $4 < x < 10$

10 Bearings with the cosine rule

Boat P sails 15 km from a harbour on a bearing of 075°.
Boat Q sails 25 km from the same harbour on a bearing of 290°. (Calculator)
Work out the distance between the two boats.
Give your answer to 3 significant figures.

The angle between the two paths = $360° - |75° - 290°| = 145°$

$d^2 = 15^2 + 25^2 - 2 \times 15 \times 25 \times \cos 145°$

$d^2 = 1464.4$

$d = 38.3$

Answer: 38.3 km