Day 167 **A Tier 10-a-day: Model Solutions**

1 Median and interquartile range

Here are the ages, in years, of 7 people.
20 74 66 50 67 47 42
(a) Work out the median.
(b) Work out the interquartile range.

Put in order: 20, 42, 47, 50, 66, 67, 74
(a) The middle value is number 4, so the median is 50
(b) Lower half: 20, 42, 47, so LQ = 42
Upper half: 66, 67, 74, so UQ = 67
IQR = 67 − 42 = 25
Answer: (a) 50 (b) 25

2 Prime factors

Express 2156 as a product of prime factors. Give your answer in index form.

Divide by prime numbers in turn: 2156 ÷ 2 = 1078, 1078 ÷ 2 = 539, 539 ÷ 7 = 77, 77 ÷ 7 = 11, 11 ÷ 11 = 1
2156 = 2 × 2 × 7 × 7 × 11
In index form: $2156 = 2^2 \times 7^2 \times 11$
Answer: $2156 = 2^2 \times 7^2 \times 11$

3 Volume of a hollow cylinder

The diagram shows the cross-section of a metal pipe.
The outer radius is 7 cm and the inner radius is 3 cm.
The pipe is 40 cm long.
Work out the volume of metal used. Give your answer in terms of π.

Area of the ring = $\pi \times 7^2 - \pi \times 3^2 = \pi(49 - 9) = 40\pi$
Volume = area × length = $40\pi \times 40$
$= 1600\pi$
Answer: 1600π cm^3

4 Estimation

By rounding each number to 1 significant figure, estimate the value of

$$\frac{78.0 \times 50}{0.45}$$

Round each number to 1 significant figure: 80, 50, 0.5

$$\frac{80 \times 50}{0.5} = 8000$$

Answer: 8000

5 Rearranging to solve quadratics

Solve $x^2 = 6x - 5$

Move everything to one side: $x^2 - 6x + 5 = 0$
Factorise: $(x - 5)(x - 1) = 0$
$x = 1$ or $x = 5$
Answer: $x = 1$ or $x = 5$

6 Area of shaded regions with circles

3 identical circles of radius 2 cm fit exactly inside a rectangle.
The circles touch each other and the sides of the rectangle. (Calculator)
Work out the area of the shaded region.
Give your answer to 3 significant figures.

Rectangle: $12 \times 4 = 48$ cm^2
3 circles: $3 \times \pi \times 2^2 = 12\pi$
Shaded area = $48 - 12\pi = 10.3$
Answer: 10.3 cm^2

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7 Square and cube numbers

n is the smallest positive integer such that 30 × n is a square number.
Find the value of n.

Write 30 as a product of prime factors: 30 = 2 × 3 × 5
For a square number every prime must have a power that is a multiple of 2.
Extra primes needed: 2 (needs 1 more), 3 (needs 1 more), 5 (needs 1 more)
n = 30
Answer: n = 30

8 Forming and solving equations

Tom is x years old.
His dad is 3 times as old as Tom, plus 2 years.
The sum of their ages is 30 years.
Work out how old his dad is.

Let Tom's age be x, so his dad's age is 3x + 2
x + (3x + 2) = 30
4x = 28
x = 7
His dad's age = 3 × 7 + 2 = 23
Answer: 23 years

9 Inverse proportion formulae

y is inversely proportional to x^2.
When x = 8, y = 8.
(a) Find a formula for y in terms of x.
(b) Work out the value of y when x = 2.

(a) $y = \frac{k}{x^2}$. Substitute: $8 = \frac{k}{64}$, so k = 8 × 64 = 512

$y = \frac{512}{x^2}$

(b) $y = \frac{512}{4} = 128$

Answer: (a) $\mathbf{y = \frac{512}{x^2}}$ **(b) 128**

10 Simultaneous equations (one quadratic)

Solve the simultaneous equations
$x^2 + y^2 = 25$
$y = -x - 7$

Substitute $y = -x - 7$ into $x^2 + y^2 = 25$
$x^2 + (-x - 7)^2 = 25$
$2x^2 + 14x + 24 = 0$
Divide by 2: $x^2 + 7x + 12 = 0$
(x + 4)(x + 3) = 0
x = −4 or x = −3
Use $y = -x - 7$ to find y: when x = −4, y = −3; when x = −3, y = −4
Answer: x = −4, y = −3 and x = −3, y = −4