Day 164 **A Tier 10-a-day: Model Solutions**

1 Volume of a cylinder

A tin of soup is a cylinder with radius 7 cm and height 19 cm.
Work out the volume. Give your answer in terms of π.

Volume = $\pi r^2 h$
$= \pi \times 7^2 \times 19$
$= 931\pi$
Answer: 931π cm^3

2 Roots and powers

(a) Between which two consecutive whole numbers does $\sqrt[3]{88}$ lie?
(b) Work out $3^3 - 4^2$

(a) $4^3 = 64$ and $5^3 = 125$, and 88 is between these.
(b) $3^3 = 27$ and $4^2 = 16$, so $27 - 16 = 11$
Answer: (a) 4 and 5 (b) 11

3 Mean from a frequency table

Here is some information about 20 matches.
(a) Work out the mean.
(b) Write down the mode.

(a) Total = $0 \times 2 + 1 \times 2 + 2 \times 5 + 3 \times 9 + 4 \times 2 = 47$
Mean = $47 \div 20 = 2.35$
(b) The highest frequency is 9, so the mode is 3
Answer: (a) 2.35 (b) 3

4 Area of a trapezium

The area of this trapezium is 168 cm^2.
Work out the value of x.
The diagram is not drawn accurately.

$\frac{1}{2} \times (x + 19) \times 12 = 168$
$(x + 19) \times 12 = 336$
$x + 19 = 336 \div 12 = 28$
$x = 9$
Answer: x = 9

5 Difference of two squares

(a) Factorise $4y^2 - 25z^2$
(b) Use the same idea to work out $36^2 - 14^2$ without a calculator.

(a) $4y^2 - 25z^2 = (2y)^2 - (5z)^2$
$(2y - 5z)(2y + 5z)$
(b) $a^2 - b^2 = (a + b)(a - b)$, so $36^2 - 14^2 = (36 + 14)(36 - 14)$
$= 50 \times 22 = 1100$
Answer: (a) $(2y - 5z)(2y + 5z)$ (b) 1100

6 Recurring decimals to fractions

Write the recurring decimal $0.\dot{6}\dot{7}$ as a fraction in its simplest form.

Let x = 0.676767...
Multiply by 100: 100x = 67.6767...
Subtract: 99x = 67
$x = \frac{67}{99}$
Answer: $\frac{67}{99}$

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7 Algebraic rearrangement and change of sign

(a) Show that the equation $x^3 + 6x - 37 = 0$ can be rearranged to give $x = \sqrt[3]{37 - 6x}$
(b) Show that the equation has a solution between x = 2 and x = 3

(a) Add 37 and subtract 6x from both sides: $x^3 = 37 - 6x$
Take the cube root of both sides: $x = \sqrt[3]{37 - 6x}$
(b) Let $f(x) = x^3 + 6x - 37$
$f(2) = -17$
$f(3) = 8$
There is a change of sign (negative to positive) and f is continuous, so there is a solution between x = 2 and x = 3
Answer: Shown: f(2) = −17, f(3) = 8 (change of sign)

8 Probabilities adding to 1

The table shows the probabilities of the outcomes of a biased spinner.
(a) Work out the value of y.
(b) The spinner is spun 100 times. Estimate the number of times it lands on grey.

(a) Probabilities add to 1: 0.35 + y + 2y + 0.35 = 1
0.7 + 3y = 1, so 3y = 0.3
y = 0.1
(b) P(grey) = 0.2, so expected number = 0.2 × 100 = 20
Answer: (a) y = 0.1 (b) 20

9 Reverse compound change

The value of a house increases by 20% each year. (Calculator)
After 2 years it is worth £29,520.
Work out its value at the start.

Yearly multiplier = 1.2
Overall multiplier = $1.2^2 = 1.44$
Original value = £29,520 ÷ 1.44 = £20,500
Answer: £20,500

10 Probability with three selections

A bag contains 4 mint sweets and 5 toffee sweets.
Three sweets are taken at random without replacement.
Work out the probability that all three sweets are mint.

$P(\text{all mint}) = \frac{4}{9} \times \frac{3}{8} \times \frac{2}{7}$

$= \frac{24}{504} = \frac{1}{21}$

Answer: $\frac{1}{21}$