Day 163 **A Tier 10-a-day: Model Solutions**

1 Exact trigonometric values

Without using a calculator, work out the exact value of
$3 \tan 45° - 5 \cos 30°$
Give your answer in its simplest form.

$\tan 45° = 1$ and $\cos 30° = \frac{\sqrt{3}}{2}$

$3 \times 1 - 5 \times \frac{\sqrt{3}}{2} = \frac{6 - 5\sqrt{3}}{2}$

Answer: $\mathbf{\frac{6 - 5\sqrt{3}}{2}}$

2 Iteration

$x_{n+1} = \frac{8}{(x_n + 5)}$
$x_0 = 3$
Work out the values of x_1, x_2 and x_3.
Give your answers correct to 3 decimal places. (Calculator)

Substitute $x_0 = 3$ into the formula, then use each answer in the next step (keep the full calculator display)
$x_1 = 1$
$x_2 = 1.33333$
$x_3 = 1.26316$
Answer: $x_1 = 1.000$, $x_2 = 1.333$, $x_3 = 1.263$

3 Reverse percentages

A bicycle costs £960 including VAT at 20%.
Work out the price of the bicycle before VAT was added.

120% represents £960
1% = £960 ÷ 120 = £8
100% = £8 × 100 = £800.00
Answer: £800.00

4 Pythagoras' theorem

Work out the length of x.
The diagram is not drawn accurately.

$15^2 = x^2 + 9^2$
$x^2 = 225 - 81 = 144$
$x = \sqrt{144} = 12$
Answer: x = 12 cm

5 Simplifying surds

(a) Simplify $\sqrt{90}$
(b) Simplify $\sqrt{90} + \sqrt{90}$

(a) $\sqrt{90} = \sqrt{9} \times \sqrt{10} = 3\sqrt{10}$
(b) $\sqrt{90} = 3\sqrt{10}$, so $3\sqrt{10} + 3\sqrt{10} = 6\sqrt{10}$
Answer: (a) $3\sqrt{10}$ (b) $6\sqrt{10}$

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6 Similar triangles with parallel lines

In triangle ABC, DE is parallel to BC.
D is on AB and E is on AC.
Work out the value of x.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar (DE is parallel to BC)

AE ÷ AC = AD ÷ AB, so AC = $14 \times \frac{9}{7} = 18$

x = EC = 18 − 14 = 4

Answer: x = 4

7 Surd fractions

Show that $\frac{\sqrt{108} - \sqrt{27}}{\sqrt{3}}$ is an integer, and find its value.

$\sqrt{108} = 6\sqrt{3}$ and $\sqrt{27} = 3\sqrt{3}$

Numerator = $6\sqrt{3} - 3\sqrt{3} = 3\sqrt{3}$

$\frac{3\sqrt{3}}{\sqrt{3}} = 3$

Answer: 3

8 Percentages in context

Grace earns £25000 a year.
She pays no tax on the first £15000.
She pays 30% tax on the rest.
Work out how much tax she pays in a year.

Taxable amount = £25000 − £15000 = £10000

Tax = 30% of £10000 = £3000

Answer: £3000

9 Frequency density

The table gives information about 151 measurements.
A histogram is drawn using this information.
(a) Work out the missing frequency.
(b) Work out the height of the bar for the class $45 < h \leq 75$.

(a) Missing frequency = 151 − (9 + 16 + 78) = 48

(b) Class width = 75 − 45 = 30

Frequency density = 78 ÷ 30 = 2.6

Answer: (a) 48 (b) 2.6

10 Expanding three brackets

Expand and simplify
$(x - 1)(x + 5)(x - 5)$

Expand the first two brackets: $(x - 1)(x + 5) = x^2 + 4x - 5$

Multiply by the third bracket: $(x^2 + 4x - 5)(x - 5)$

$= x^3 + 4x^2 - 5x - 5x^2 - 20x + 25$

$= x^3 - x^2 - 25x + 25$

Answer: $x^3 - x^2 - 25x + 25$