Day 160 **A Tier 10-a-day: Model Solutions**

1 Laws of indices

Simplify fully

(a) $\frac{6x^2y^2}{3xy^6}$

(b) $(3x^3y^3)^2$

(a) Divide the numbers: $6 \div 3 = 2$

Subtract the powers: $x^2 \div x = x$ and $y^2 \div y^6 = y^{-4}$

$= \frac{2x}{y^4}$

(b) Raise every part to the power 2: $3^2 = 9$, $(x^3)^2 = x^6$, $(y^3)^2 = y^6$

$= 9x^6y^6$

Answer: (a) $\frac{2x}{y^4}$ (b) $9x^6y^6$

2 Map scales

The scale of a map is 1 : 20000.
Two towns are 2 cm apart on the map.
Work out the real distance between the towns in kilometres.

Real distance = $2 \times 20000 = 40000$ cm

40000 cm = 400 m = 0.4 km

Answer: 0.4 km

3 Area of shaded regions with circles

4 identical circles of radius 7 cm fit exactly inside a rectangle.
The circles touch each other and the sides of the rectangle. (Calculator)
Work out the area of the shaded region.
Give your answer to 3 significant figures.

Rectangle: $56 \times 14 = 784$ cm^2

4 circles: $4 \times \pi \times 7^2 = 196\pi$

Shaded area = $784 - 196\pi = 168$

Answer: 168 cm^2

4 Factorising

Factorise $10x^2 - 7x - 6$

Multiply $10 \times -6 = -60$. Find two numbers that multiply to -60 and add to -7: 5 and -12

$10x^2 - 7x - 6 = 10x^2 + 5x - 12x - 6$

Factorise in pairs to get $(5x - 6)(2x + 1)$

Answer: $(5x - 6)(2x + 1)$

5 Growth and decay

A house is worth £250000. (Calculator)
Its value increases by 12% each year.
Work out its value after 4 years., to the nearest pound.

Multiplier = 100% + 12% = 1.12

$250000 \times 1.12^4 = 393379.84$

= £393380 to the nearest whole number

Answer: £393380

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6 Circle theorems: angle at the centre

X, Y and Z are points on a circle with centre O.
Angle XZY = 54°.
Work out the size of angle x.
Give a reason for your answer.

The angle at the centre is twice the angle at the circumference
x = 2 × 54 = 108
Answer: x = 108°

7 Cumulative frequency graphs

The cumulative frequency graph shows information about the age of 100 visitors.
Use the graph to find an estimate for
(a) the median,
(b) the number of visitors with a age greater than 10 years.

(a) Median at cumulative frequency 100 ÷ 2 = 50
Read across from 50 to the graph, then down to the axis
Median = 15 years
(b) At 10 years the cumulative frequency is 43
100 − 43 = 57
Answer: (a) 15 years (b) 57

8 Recurring decimals, show that

(a) Show that $0.\dot{7}\dot{5} = \frac{25}{33}$

(b) Hence write $2.\dot{7}\dot{5}$ as a mixed number in its simplest form.

(a) Let x = 0.757575..., so 100x = 75.7575...

100x − x = 75, so 99x = 75 and $x = \frac{75}{99} = \frac{25}{33}$

(b) $2.\dot{7}\dot{5} = 2 + \frac{25}{33} = 2\frac{25}{33}$

Answer: (b) $\mathbf{2\frac{25}{33}}$

9 Simplifying surds

(a) Simplify $\sqrt{108}$

(b) Simplify $\sqrt{3} \times \sqrt{27}$

(a) $\sqrt{108} = \sqrt{36} \times \sqrt{3} = 6\sqrt{3}$
(b) $\sqrt{3} \times \sqrt{27} = \sqrt{81} = 9$
Answer: (a) $\mathbf{6\sqrt{3}}$ **(b) 9**

10 Angles in regular polygons around a point

A regular decagon and a regular dodecagon share the side AB and are on opposite sides of it.
The two shapes meet at the point A.
Work out the size of the angle at A that is not inside either shape.

Interior angle of the regular decagon = 180 − 360 ÷ 10 = 144°
Interior angle of the regular dodecagon = 180 − 360 ÷ 12 = 150°
Angles round a point add up to 360°
360 − 144 − 150 = 66
Answer: 66°