Day 16 **A Tier 10-a-day: Model Solutions**

1 Isosceles triangles and exterior angles

LMN is an isosceles triangle with LM = LN.
MNK is a straight line.
Angle LNK = 109°.
Work out the size of angle x.
The diagram is not drawn accurately.

Angles on a straight line add up to 180°
Angle LNM = 180 − 109 = 71°
Base angles are equal, so angle LMN = 71°
x = 180 − 71 − 71 = 38
Answer: x = 38°

2 Algebraic formulae in context

A hall is hired for £50 plus £25 for each hour.
(a) Write a formula for the total cost, £C, after h hours.
(b) Maria pays £400 in total. Work out the number of hours.

(a) C = 50 + 25h
(b) 50 + 25h = 400
25h = 350
h = 14
Answer: (a) C = 50 + 25h (b) 14

3 Standard form notation

(a) Write 0.000051 in standard form.
(b) Write 5.07×10^4 as an ordinary number.

(a) Move the decimal point 5 places to get 5.1: 5.1×10^{-5}
(b) Move the decimal point 4 places to the right: 50,700
Answer: (a) 5.1×10^{-5} (b) 50,700

4 The quadratic formula

Solve $x^2 + 9x - 3 = 0$
Give your solutions correct to 2 decimal places. (Calculator)

Use $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ with a = 1, b = 9, c = −3

$x = \frac{-9 \pm \sqrt{9^2 - 4 \times 1 \times (-3)}}{2}$

$x = \frac{-9 \pm \sqrt{93}}{2}$

x = 0.32 or x = −9.32
Answer: x = 0.32 or x = −9.32

5 Probabilities adding to 1

The table shows the probabilities of the outcomes of a biased four-sided dice.
(a) Work out the value of p.
(b) The four-sided dice is rolled 100 times. Estimate the number of times it lands on 2.

(a) Probabilities add to 1: 0.15 + p + p + 0.45 = 1
0.6 + 2p = 1, so 2p = 0.4
p = 0.2
(b) P(2) = 0.2, so expected number = 0.2 × 100 = 20
Answer: (a) p = 0.2 (b) 20

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6 Rationalising the denominator

Rationalise the denominator of $\frac{7}{\sqrt{12}}$

Give your answer in its simplest form.

Simplify the denominator: $\sqrt{12} = 2\sqrt{3}$

$\frac{7}{\sqrt{12}} = \frac{7}{2\sqrt{3}}$

Multiply top and bottom by $\sqrt{3}$: $\frac{7\sqrt{3}}{2 \times 3}$

$= \frac{7\sqrt{3}}{6}$

Answer: $\mathbf{\frac{7\sqrt{3}}{6}}$

7 Combined means

The mean age of 7 workers is 61 years.
One of them, aged 49, leaves.
Work out the mean age of the remaining workers.

Total age = $7 \times 61 = 427$
Total after one leaves = $427 - 49 = 378$
Mean = $378 \div 6 = 63$
Answer: 63 years

8 Circles and tangents

The point P(8, −15) lies on a circle with centre O(0, 0).
(a) Work out the equation of the circle, in the form $x^2 + y^2 = r^2$.
(b) Find the equation of the tangent to the circle at P.

(a) The circle is $x^2 + y^2 = r^2$, so $r^2 = 8^2 + (-15)^2 = 289$

(b) Gradient of the radius OP = $\frac{-15}{8}$

The tangent is perpendicular to the radius, so its gradient = $-1 \div (-\frac{15}{8}) = \frac{8}{15}$

$y = \frac{8}{15}x + c$ through (8, −15): $-15 = \frac{8}{15} \times 8 + c$, so $c = -\frac{289}{15}$

$y = \frac{8}{15}x - \frac{289}{15}$

Answer: (a) $\mathbf{x^2 + y^2 = 289}$ **(b)** $\mathbf{y = \frac{8}{15}x - \frac{289}{15}}$

9 Perimeter and area with algebra

An isosceles triangle has two equal sides of length (4x + 4) cm and (3x + 12) cm.
The third side is 14 cm.
Work out the perimeter of the triangle.

The two equal sides give an equation
$4x + 4 = 3x + 12$
$x = 8$
$x = 8$
Equal sides = $4 \times 8 + 4 = 36$ cm
Perimeter = $36 + 36 + 14 = 86$
Answer: 86 cm

10 Finding a rate of compound change

Hassan invests £7,500 in a savings account. (Calculator)
Compound interest is paid at a fixed rate each year.
After 2 years the investment is worth £8,112.
Work out the annual rate of interest.

Overall multiplier = £8,112 ÷ £7,500 = 1.0816

Take the square root: $\sqrt{1.0816} = 1.04$

Rate = (1.04 − 1) × 100 = 4%

Answer: 4%