Day 158 **A Tier 10-a-day: Model Solutions**

1 Circle area and circumference

The area of a circle is 25π cm^2.
(a) Work out the radius of the circle.
(b) Work out the circumference. Give your answer in terms of π.

(a) $\pi r^2 = 25\pi$, so $r^2 = 25$
$r = 5$
(b) Circumference $= 2\pi r = 2 \times \pi \times 5 = 10\pi$
Answer: (a) 5 cm (b) 10π cm

2 Two dice

Two fair dice are thrown.
The two scores are multiplied together.
Work out the probability that the product is even.

Easier to count the outcomes that do not work: (1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5)
That is 9 outcomes, so 27 work
$\frac{27}{36} = \frac{3}{4}$

Answer: $\frac{3}{4}$

3 nth term of a linear sequence

Here are the first four terms of a sequence.
2, 6, 10, 14
(a) Find an expression for the nth term.
(b) Find the first term in the sequence that is greater than 100.

The terms go up by 4 each time, so the nth term starts with $4n$
$2 - 4 = -2$, so the nth term is $4n - 2$
(b) $4n - 2 > 100$
$4n > 102$
$n > \frac{51}{2}$
The smallest whole number is $n = 26$, and the term is 102
Answer: (a) $4n - 2$ (b) 102

4 Percentage change

Ali buys a second-hand car for £150.
He sells it for £97.50.
Work out his percentage loss.

Loss = £150.00 − £97.50 = £52.50
Percentage loss $= \frac{£52.50}{£150.00} \times 100 = 35\%$
Answer: 35%

5 Volume of a prism

The diagram shows a prism with a right-angled triangle as its cross-section.
The volume of the prism is 475 cm^3.
Work out the value of x.
The diagram is not drawn accurately.

Area of cross-section $= \frac{1}{2} \times 5 \times 10 = 25$
Volume = area × length
$475 = 25 \times$ length
length $= 475 \div 25 = 19$
Answer: x = 19

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6 Combining ratios

a : b = 1 : 2
b : c = 5 : 1
(a) Work out the ratio a : b : c
(b) a, b and c add up to 102. Work out the value of c.

(a) Make the b parts equal: lowest common multiple of 2 and 5 is 10
a : b = 5 : 10 and b : c = 10 : 2
a : b : c = 5 : 10 : 2
(b) 5 + 10 + 2 = 17 parts, so 1 part = 102 ÷ 17 = 6
c = 2 × 6 = 12
Answer: (a) 5 : 10 : 2 (b) 12

7 Growth and decay

A car is worth £25,000 and loses 6% of its value each year. (Calculator)
Work out its value after 6 years.
Give your answer to the nearest pound.

Multiplier = 0.94
£25,000 × 0.94^6 = £17,246.74
= £17,247 to the nearest whole number
Answer: £17,247

8 Rearranging formulae (subject twice)

Make x the subject of $y = \frac{3x - 5}{-x + 5}$

Multiply both sides by $(-x + 5)$: $y(-x + 5) = 3x - 5$
Expand the bracket: $-xy + 5y = 3x - 5$
Collect the x terms on one side: $-xy - 3x = -5y - 5$
Factorise: $x(-y - 3) = -5y - 5$
Divide by $(-y - 3)$: $x = \frac{-5y - 5}{-y - 3}$
Multiply the top and bottom by -1: $x = \frac{5y + 5}{y + 3}$
Answer: $x = \frac{5y + 5}{y + 3}$

9 Exact trigonometric values

PQR is a right-angled triangle with the right angle at Q.
PR = 6 cm and angle QPR = 45°.
Work out the exact length of QR.
Do not use a calculator.

QR is opposite the 45° angle and PR is the hypotenuse, so use sin
$\sin 45° = \frac{\sqrt{2}}{2}$
$QR = 6 \times \frac{\sqrt{2}}{2} = 3\sqrt{2}$
Answer: $3\sqrt{2}$ cm

10 Finding a rate of compound change

Oscar invests £5,000 in a savings account. (Calculator)
Compound interest is paid at a fixed rate each year.
After 2 years the investment is worth £6,050.
Work out the annual rate of interest.

Overall multiplier = £6,050 ÷ £5,000 = 1.21
Take the square root: $\sqrt{1.21} = 1.1$
Rate = (1.1 − 1) × 100 = 10%
Answer: 10%