Day 157 **A Tier 10-a-day: Model Solutions**

1 Loci: area of a region

A horse is tied by a rope of length 4 m to the corner of a rectangular shed.
The shed measures 10 m by 6 m.
The horse can graze on any grass it can reach outside the shed.
Work out the area it can graze. Give your answer in terms of π.
The diagram is not drawn accurately.

The rope can sweep a sector of angle $360° - 90° = 270°$

Area $= \frac{270}{360} \times \pi \times 4^2$

$= \frac{3}{4} \times 16\pi$

$= 12\pi$

Answer: 12π m^2

2 Prime factors

Write 1350 as a product of prime factors, in index form.

Divide by prime numbers in turn: $1350 \div 2 = 675$, $675 \div 3 = 225$, $225 \div 3 = 75$, $75 \div 3 = 25$, $25 \div 5 = 5$, $5 \div 5 = 1$
$1350 = 2 \times 3 \times 3 \times 3 \times 5 \times 5$
In index form: $1350 = 2 \times 3^3 \times 5^2$
Answer: $1350 = 2 \times 3^3 \times 5^2$

3 Inverse proportion

7 painters take 22 days to paint a hall.
All the painters work at the same rate.
How many painters are needed to paint a hall in 77 days?

Total work $= 7 \times 22 = 154$ painter-day
Number needed $= 154 \div 77 = 2$
Answer: 2

4 Converting units of area and volume

Convert 20000 cm^2 into m^2.

1 m^2 = 10 000 cm^2
$20000 \div 10\,000 = 2$
Answer: 2 m^2

5 Solving quadratic equations

Solve $2x^2 - 7x - 4 = 0$

Factorise: $(x - 4)(2x + 1) = 0$
$x - 4 = 0$ or $2x + 1 = 0$
$x = 4$ or $x = -\frac{1}{2}$
Answer: $x = 4$ or $x = -\frac{1}{2}$

6 Probabilities adding to 1

The table shows the probabilities of the outcomes of a biased spinner.
(a) Work out the value of y.
(b) The spinner is spun 300 times. Estimate the number of times it lands on white.

(a) Probabilities add to 1: $0.25 + y + 2y + 0.45 = 1$
$0.7 + 3y = 1$, so $3y = 0.3$
$y = 0.1$
(b) P(white) = 0.1, so expected number $= 0.1 \times 300 = 30$
Answer: (a) y = 0.1 (b) 30

Day 157 **A Tier 10-a-day: Model Solutions**

7 Equations with brackets

Solve $-3(x - 1) = 4(x - 4) - 9$

Expand the brackets: $-3x + 3 = 4x - 16 - 9$
Simplify: $-3x + 3 = 4x - 25$
Collect x terms on one side and numbers on the other: $-7x = -28$
$x = 4$
Answer: x = 4

8 Rationalising the denominator

Rationalise the denominator of $\frac{18}{\sqrt{27}}$

Give your answer in its simplest form.

Simplify the denominator: $\sqrt{27} = 3\sqrt{3}$
$\frac{18}{\sqrt{27}} = \frac{18}{3\sqrt{3}}$

Multiply top and bottom by $\sqrt{3}$: $\frac{18\sqrt{3}}{3 \times 3}$

$= 2\sqrt{3}$
Answer: $2\sqrt{3}$

9 Compound shapes with circles

The diagram shows a shape made from a rectangle and a semicircle.
Work out the perimeter of the shape.
Give your answer in terms of π.
The diagram is not drawn accurately.

Curved edge: $\frac{1}{2} \times \pi \times 12 = 6\pi$
Straight edges: $12 + 5 + 5 = 22$
Total = $22 + 6\pi$
Answer: $22 + 6\pi$ cm

10 Two dice

Two fair dice are thrown.
The two scores are multiplied together.
Work out the probability that the product is greater than 3.

Easier to count the outcomes that do not work: (1, 1), (1, 2), (1, 3), (2, 1), (3, 1)
That is 5 outcomes, so 31 work
$\frac{31}{36}$
Answer: $\frac{31}{36}$