Day 156 **A Tier 10-a-day: Model Solutions**

1 nth term of a linear sequence

Here are the first four terms of a sequence.
14, 12, 10, 8
(a) Find an expression for the nth term.
(b) Find the first term in the sequence that is less than −150.

The terms change by −2 each time, so the nth term starts with −2n
14 − (−2) = 16, so the nth term is −2n + 16
(b) −2n + 16 < −150
−2n < −166
Divide by −2 and reverse the inequality sign: n > 83
The smallest whole number is n = 84, and the term is −152
Answer: (a) −2n + 16 (b) −152

2 Standard form in context

Light travels at 3×10^5 kilometres per second.
A distant star is 8.4×10^9 kilometres from Earth.
Work out how many seconds light takes to travel from a distant star to Earth.
Give your answer in standard form.

time = distance ÷ speed = $(8.4 \times 10^9) \div (3 \times 10^5)$
$8.4 \div 3 = 2.8$ and $10^9 \div 10^5 = 10^4$
$= 2.8 \times 10^4$ seconds
Answer: 2.8×10^4 seconds

3 Circle theorems: angle in a semicircle

XY is a diameter of the circle.
Angle YXZ = 25°.
Work out the size of angle x.
Give a reason for each step of your working.

The angle in a semicircle is 90°, so angle XZY = 90°
Angles in a triangle add up to 180°
x = 180 − 90 − 25 = 65
Answer: x = 65°

4 Expanding double brackets

Expand and simplify
(3x − 5)(2x + 2)

Multiply every term in the first bracket by every term in the second
$= 6x^2 + 6x - 10x - 10$
$= 6x^2 - 4x - 10$
Answer: $6x^2 - 4x - 10$

5 Tree diagrams

The tree diagram shows probabilities for two events.
(a) Work out the values of a and b.
(b) Work out the probability that Leah buys an ice cream.

(a) Probabilities on pairs of branches add up to 1.
a = 1 − 0.1 = 0.9 and b = 1 − 0.4 = 0.6
(b) P(ice cream) = 0.1 × 0.4 + 0.9 × 0.6
= 0.04 + 0.54 = 0.58
Answer: (a) a = 0.9, b = 0.6 (b) 0.58

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6 Surface area of a cuboid

A closed gift box is a cuboid measuring 5 cm by 9 cm by 5 cm.
Work out the total surface area.

Faces: 5 × 9, 5 × 5 and 9 × 5 each appear twice
2 × (45 + 25 + 45)
= 2 × 115 = 230
Answer: 230 cm^2

7 Reverse compound change

The value of an investment grows by 2% each year. (Calculator)
After 2 years it is worth £31,212.
Work out its value at the start.

Yearly multiplier = 1.02
Overall multiplier = 1.02^2 = 1.0404
Original value = £31,212 ÷ 1.0404 = £30,000
Answer: £30,000

8 Quadratic inequalities

Solve $x^2 - 5x - 14 < 0$

Solve $x^2 - 5x - 14 = 0$: $(x + 2)(x - 7) = 0$
$x = -2$ or $x = 7$
Sketch the U-shaped curve $y = x^2 - 5x - 14$, crossing the x-axis at -2 and 7
The curve is below the x-axis between the roots
$-2 < x < 7$
Answer: $-2 < x < 7$

9 Congruent triangles

Triangle UVW: UV = 11 cm, VW = 4 cm, UW = 13 cm.
Triangle ABC: AB = 11 cm, BC = 4 cm, AC = 14 cm.
Are the two triangles congruent?
Give a reason for your answer.

The sides of triangle UVW are 11, 4 and 13
The sides of triangle ABC are 11, 4 and 14
The side lengths are not the same, so the triangles are not congruent
Answer: No, not congruent

10 Probability with three selections

A bag contains 3 mint sweets and 6 toffee sweets.
Three sweets are taken at random without replacement.
Work out the probability that exactly one of the three sweets is toffee.

One order, for example toffee, mint, mint: $\frac{6}{9} \times \frac{3}{8} \times \frac{2}{7} = \frac{1}{14}$
There are 3 possible orders, each with the same probability
$3 \times \frac{1}{14} = \frac{3}{14}$
Answer: $\frac{3}{14}$