Day 152 **A Tier 10-a-day: Model Solutions**

1 Angles in parallel lines

The diagram shows two parallel lines crossed by a straight line.
(a) Work out the value of x.
(b) Work out the size of the angle marked (5x − 20)°.
The diagram is not drawn accurately.

(a) Alternate angles are equal
5x − 20 = x + 44
4x = 64
x = 16
(b) 5 × 16 − 20 = 60
Answer: (a) x = 16 (b) 60°

2 Mean from a frequency table

The table shows information about 20 students.
(a) Work out the mean.
(b) Write down the mode.

(a) Total = 0 × 2 + 1 × 1 + 2 × 7 + 3 × 6 + 4 × 4 = 49
Mean = 49 ÷ 20 = 2.45
(b) The highest frequency is 7, so the mode is 2
Answer: (a) 2.45 (b) 2

3 Powers and roots in standard form

Work out $\sqrt{4 \times 10^{-10}}$
Give your answer in standard form.

$\sqrt{4 \times 10^{-10}} = \sqrt{4} \times \sqrt{10^{-10}}$
$= 2 \times 10^{-5}$
Answer: 2×10^{-5}

4 Angles in regular polygons around a point

A regular octagon and a regular hexagon share the side AB and are on opposite sides of it.
The two shapes meet at the point A.
Work out the size of the angle at A that is not inside either shape.

Interior angle of the regular octagon = 180 − 360 ÷ 8 = 135°
Interior angle of the regular hexagon = 180 − 360 ÷ 6 = 120°
Angles round a point add up to 360°
360 − 135 − 120 = 105
Answer: 105°

5 Inverse proportion

11 pumps take 6 hours to empty a pond.
All the pumps work at the same rate.
How long would it take 3 pumps to empty a pond?

Total work = 11 × 6 = 66 pump-hour
Time for 3 pumps = 66 ÷ 3 = 22 hours
Answer: 22 hours

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6 Forming quadratic equations

A rectangle has width x cm and length (x + 7) cm.
The area of the rectangle is 98 cm^2.
(a) Show that $x^2 + 7x - 98 = 0$
(b) Work out the value of x.

(a) Area = length × width, so $x(x + 7) = 98$
$x^2 + 7x = 98$
$x^2 + 7x - 98 = 0$
(b) Factorise: $(x - 7)(x + 14) = 0$
$x = 7$ or $x = -14$
A length cannot be negative, so $x = 7$
Answer: (a) Shown (b) x = 7

7 Changing a ratio

A bag contains red counters and blue counters in the ratio 5 : 4.
14 more red counters are added to the bag.
The ratio of red counters to blue counters is now 8 : 5.
How many counters were in the bag at the start?

Let red = 5x and blue = 4x. After adding: $(5x + 14) : 4x = 8 : 5$
$\frac{5x + 14}{4x} = \frac{8}{5}$, so $5(5x + 14) = 8 \times 4x$
$25x + 70 = 32x$, so $7x = 70$, $x = 10$
Start: $5x + 4x = 9 \times 10 = 90$ counters
Answer: 90

8 Recurring decimals, show that

(a) Show that $0.\dot{3}\dot{2} = \frac{32}{99}$

(b) Hence write $5.\dot{3}\dot{2}$ as a mixed number in its simplest form.

(a) Let x = 0.323232..., so 100x = 32.3232...
$100x - x = 32$, so $99x = 32$ and $x = \frac{32}{99}$
(b) $5.\dot{3}\dot{2} = 5 + \frac{32}{99} = 5\frac{32}{99}$
Answer: (b) $\mathbf{5\frac{32}{99}}$

9 Adding and subtracting in standard form

Work out $(6.5 \times 10^{-2}) - (2.4 \times 10^{-3})$
Give your answer in standard form.

Write both numbers using 10^{-3}: $6.5 \times 10^{-2} = 65 \times 10^{-3}$
$(65 - 2.4) \times 10^{-3} = 62.6 \times 10^{-3}$
In standard form: 6.26×10^{-2}
Answer: $\mathbf{6.26 \times 10^{-2}}$

10 Rearranging formulae (subject twice)

Make x the subject of $y = \frac{3x + 6}{x - 4}$

Multiply both sides by $(x - 4)$: $y(x - 4) = 3x + 6$
Expand the bracket: $xy - 4y = 3x + 6$
Collect the x terms on one side: $xy - 3x = 4y + 6$
Factorise: $x(y - 3) = 4y + 6$
Divide by $(y - 3)$: $x = \frac{4y + 6}{y - 3}$
Answer: $\mathbf{x = \frac{4y + 6}{y - 3}}$