Day 151 **A Tier 10-a-day: Model Solutions**

1 Circle area and circumference

A circular pond has radius 2 m.
Work out, in terms of π,
(a) the circumference,
(b) the area.

(a) Circumference = $\pi d = \pi \times 4 = 4\pi$
(b) Area = $\pi r^2 = \pi \times 2^2 = 4\pi$
Answer: (a) 4π m (b) 4π m^2

2 Simultaneous equations

Solve the simultaneous equations
$-6x + 2y = -18$
$-4x + 4y = -12$

Multiply (1) by 2 to make the y coefficients match
$-12x + 4y = -36$ (1)
$-4x + 4y = -12$ (2)
The y terms have the same sign, so subtract equation (1) from equation (2)
$8x = 24$, so $x = 3$
Substitute $x = 3$ into equation (1) to find $y = 0$
Answer: x = 3, y = 0

3 Converting units of area and volume

Convert 15000 mm^2 into cm^2.

1 cm = 10 mm, so 1 cm^2 = 10 × 10 = 100 mm^2
15000 ÷ 100 = 150
Answer: 150 cm^2

4 HCF and LCM

(a) Write 72 and 60 as products of prime factors.
(b) Find the highest common factor (HCF) and the lowest common multiple (LCM) of 72 and 60.

(a) $72 = 2^3 \times 3^2$ and $60 = 2^2 \times 3 \times 5$
(b) HCF: multiply the prime factors in both lists, taking the smaller power: $2^2 \times 3 = 12$
LCM: take the larger power of every prime: $2^3 \times 3^2 \times 5 = 360$
Answer: (a) $72 = 2^3 \times 3^2$, $60 = 2^2 \times 3 \times 5$ (b) HCF = 12, LCM = 360

5 Midpoints and length of a line segment

M is the midpoint of AB.
A is the point (2, −2) and M is the point (4, 2).
Work out the coordinates of B.

From A to M: x changes by 2 and y changes by 4
Do the same again from M to get B
B = (4 + 2, 2 + 4) = (6, 6)
Answer: (6, 6)

6 Box plots

The box plot shows the scores of the students in Class A in a test.
(a) Write down the median and the interquartile range.
For Class B the median is 5 and the interquartile range is 20.
(b) Compare the distributions of the two classes.

(a) Median = 15
Interquartile range = 40 − 10 = 30
(b) The median for Class A is higher (15 compared with 5), so it has the higher values on average
The IQR for Class B is smaller (20 compared with 30), so its values are more consistent
Answer: (a) Median 15, IQR 30 (b) Class A has the higher median; Class B has the smaller IQR so is more consistent

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7 Bounds of a speed or density

A metal block has mass 670 g and volume 84 cm^3. The mass is measured to the nearest 10 grams and the volume to the nearest cm^3. (Calculator)
Work out the upper bound for the density of the metal in g/cm^3.
Give your answer correct to 3 significant figures.

density = mass ÷ volume
For the upper bound of density use the upper bound of mass and the lower bound of volume.
675 ÷ 83.5 = 8.08383
= 8.08 g/cm^3 (3 s.f.)
Answer: 8.08 g/cm^3

8 Direct proportion formulae

y is directly proportional to the square root of x.
When x = 4, y = 18.
(a) Find a formula for y in terms of x.
(b) Work out the value of y when x = 36.

(a) $y = k\sqrt{x}$. Substitute: 18 = k × 2, so k = 9
$y = 9\sqrt{x}$
(b) y = 9 × 6 = 54
Answer: (a) $y = 9\sqrt{x}$ (b) 54

9 Algebraic proof

Prove that the sum of the squares of two consecutive odd numbers is always 2 more than a multiple of 8.

Let the odd numbers be 2n + 1 and 2n + 3
$(2n + 1)^2 + (2n + 3)^2 = 4n^2 + 4n + 1 + 4n^2 + 12n + 9$
$= 8n^2 + 16n + 10$
$= 8(n^2 + 2n + 1) + 2 = 8(n + 1)^2 + 2$, which is 2 more than a multiple of 8
Answer: Proved

10 Fraction calculations

Work out $(1\frac{1}{6} + 1\frac{1}{6}) \times 4\frac{1}{2}$
Give your answer as a mixed number in its simplest form.

Brackets first, as improper fractions: $\frac{7}{6} + \frac{7}{6} = \frac{7}{3}$
Then multiply: $\frac{7}{3} \times \frac{9}{2} = \frac{21}{2}$
$= 10\frac{1}{2}$
Answer: $10\frac{1}{2}$