Day 150 **A Tier 10-a-day: Model Solutions**

1 Surface area of a prism

The diagram shows a triangular prism.
The cross-section is a right-angled triangle.
Work out the total surface area of the prism.
The diagram is not drawn accurately.

Hypotenuse = $\sqrt{12^2 + 16^2}$ = 20 cm

Two triangular ends: $2 \times \frac{1}{2} \times 12 \times 16 = 192$

Three rectangles: (12 + 16 + 20) × 20 = 960
Total = 192 + 960 = 1152

Answer: 1152 cm^2

2 Forming and solving equations

A rectangle has length (3x + 4) cm and width (2x − 1) cm.
The perimeter of the rectangle is 66 cm.
(a) Work out the value of x.
(b) Work out the length of the rectangle.

(a) Perimeter = 2 × (length + width), so 2(3x + 4 + 2x − 1) = 66
3x + 4 + 2x − 1 = 33 (half of the perimeter)
5x + 3 = 33
5x = 30
x = 6
(b) Length = 3 × 6 + 4 = 22 cm

Answer: (a) x = 6 (b) 22 cm

3 Prime factors

Write 126 as a product of prime factors, in index form.

Divide by prime numbers in turn: 126 ÷ 2 = 63, 63 ÷ 3 = 21, 21 ÷ 3 = 7, 7 ÷ 7 = 1
126 = 2 × 3 × 3 × 7
In index form: 126 = 2 × 3^2 × 7

Answer: 126 = 2 × 3^2 × 7

4 Rearranging formulae

Make a the subject of $b = \frac{a + 5}{6}$

Multiply both sides by 6: 6b = a + 5
Subtract 5 from both sides
a = 6b − 5

Answer: a = 6b − 5

5 Standard form in context

The population of Island M is 3.75 × 10^8.
The population of Island N is 1.5 × 10^7.
How many times bigger is the population of Island M than the population of Island N?

Divide: (3.75 × 10^8) ÷ (1.5 × 10^7)
= 3.75 ÷ 1.5 × 10^1 = 25

Answer: 25 times

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6 Stratified sampling

A school has 70 students in Year 9, 50 in Year 10 and 90 in Year 11.
A stratified sample of 21 students is to be taken.
Work out how many students should be chosen from Year 10.

Total = 70 + 50 + 90 = 210

Number from Year 10 = $\frac{50}{210} \times 21$

= 5

Answer: 5

7 Composite functions

f(x) = 2x − 7
g(x) = x^2 − 4
Solve gf(x) = 77

gf(x) = $(2x - 7)^2 - 4$
$(2x - 7)^2 - 4 = 77$, so $(2x - 7)^2 = 81$
$2x - 7 = \pm 9$
$2x - 7 = 9$ or $2x - 7 = -9$
$x = 8$ or $x = -1$
Answer: x = 8 or x = −1

8 Circle theorems: angles in the same segment

A, B, C and D are points on a circle. AC and BD meet at E.
Angle BAC = 29° and angle ACD = 27°.
Work out the size of angle x (angle AEB).
Give reasons for your working.

Angles in the same segment are equal, so angle ABD = angle ACD = 27°
In triangle ABE: angle BAE = 29° and angle ABE = 27°
x = 180 − 29 − 27 = 124
Answer: x = 124°

9 Finding an interest rate

Cara invests £2000 in a savings account. (Calculator)
The account pays compound interest at a fixed rate each year.
After 2 years there is £2121.80 in the account.
Work out the annual rate of interest.

Let the yearly multiplier be m. Then $2000 \times m^2 = 2121.8$
$m^2 = 2121.8 \div 2000 = 1.0609$
$m = \sqrt{1.0609} = 1.03$
Rate = (1.03 − 1) × 100 = 3%
Answer: 3%

10 Bounds of a speed or density

A metal block has mass 210 g and volume 187 cm^3. The mass is measured to the nearest 10 grams and the volume to the nearest cm^3. (Calculator)
Work out the upper bound for the density of the metal in g/cm^3.
Give your answer correct to 3 significant figures.

density = mass ÷ volume
For the upper bound of density use the upper bound of mass and the lower bound of volume.
215 ÷ 186.5 = 1.15282
= 1.15 g/cm^3 (3 s.f.)
Answer: 1.15 g/cm^3