Day 146 **A Tier 10-a-day: Model Solutions**

1 Median from a frequency table

The frequency table shows data for 31 students.
(a) Find the median.
(b) Work out the range.

(a) The median is the 16th value (31 + 1) ÷ 2 = 16
Cumulative frequencies: 7, 16, 17, 25, 31
The 16th value is 6
(b) Range = 9 − 5 = 4
Answer: (a) 6 (b) 4

2 Area of parallelograms and kites

The diagram shows a rhombus.
The diagonals are 22 cm and 14 cm and cross at right angles.
Work out the area of the rhombus.
The diagram is not drawn accurately.

Area = $\frac{1}{2}$ × (diagonal 1) × (diagonal 2)
= $\frac{1}{2}$ × 22 × 14
= 154
Answer: 154 cm^2

3 Exchange rates

The exchange rate is £1 = PLN 5.20. (Calculator)
Nadia changes £700 into Polish zloty and spends PLN 2,184.
Nadia changes the rest back into pounds at the same rate.
How much does Nadia get back?

Amount received: £700 × 5.2 = PLN 3,640
Amount left: PLN 3,640 − PLN 2,184 = PLN 1,456
In pounds: 1456 ÷ 5.2 = £280
Answer: £280

4 Inverse proportion

9 painters take 4 days to paint a hall.
All the painters work at the same rate.
How long would it take 4 painters to paint a hall?

Total work = 9 × 4 = 36 painter-day
Time for 4 painters = 36 ÷ 4 = 9 days
Answer: 9 days

5 Functions

f(x) = 2x + 6
g(x) = x^2 + 3
(a) Find f(4)
(b) Find fg(x). Give your answer in its simplest form.

(a) f(4) = 2 × 4 + 6 = 14
(b) fg(x) means f(g(x)), so replace x in f with x^2 + 3
fg(x) = 2(x^2 + 3) + 6
= 2x^2 + 12
Answer: (a) 14 (b) 2x^2 + 12

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6 Trigonometry: finding an angle

Work out the size of angle x. (Calculator)
Give your answer to 1 decimal place.
The diagram is not drawn accurately.

The sides given are the adjacent (6) and the hypotenuse (17), so use cos

$\cos x = \frac{6}{17}$

$x = \cos^{-1}(\frac{6}{17})$

$x = 69.3°$

Answer: x = 69.3°

7 Quadratic graphs

The curve C has equation $y = -x^2 + 4$
(a) Find the coordinates of the points where C crosses the x-axis.
(b) Find the coordinates of the turning point of C.

(a) Set $y = 0$: $-x^2 + 4 = 0$, so $x^2 - 4 = 0$ (divide by -1)
$(x + 2)(x - 2) = 0$
$x = -2$ or $x = 2$, so the graph crosses the x-axis at $(-2, 0)$ and $(2, 0)$
(b) The turning point is halfway between the roots: $x = (-2 + 2) \div 2 = 0$
$y = 4$, so the maximum point is (0, 4)
Answer: (a) (−2, 0) and (2, 0) (b) (0, 4)

8 Estimating the mean from grouped data

The table shows the masses, in kg, of 50 parcels.
(a) Write down the class that contains the median.
(b) Work out an estimate for the mean.

(a) The median is in position $(50 + 1) \div 2 = 25.5$. Cumulative frequencies: 16, 29, 31, 50
The median is in the class $20 < m \leq 40$
(b) Midpoints: 10, 30, 50, 70
$10 \times 16 + 30 \times 13 + 50 \times 2 + 70 \times 19 = 1980$
Estimate $= 1980 \div 50 = 39.6$
Answer: (a) $20 < m \leq 40$ (b) 39.6

9 Square and cube numbers

n is the smallest positive integer such that 540 × n is a square number.
Find the value of n.

Write 540 as a product of prime factors: $540 = 2^2 \times 3^3 \times 5$
For a square number every prime must have a power that is a multiple of 2.
Extra primes needed: 3 (needs 1 more), 5 (needs 1 more)
n = 15
Answer: n = 15

10 Volume of composite solids

A solid is made from a cylinder of radius 3 cm and a hemisphere on top, with the same radius.
The total volume of the solid is 126π cm^3.
Work out the height of the cylinder.

Hemisphere $= \frac{2}{3} \times \pi \times 3^3 = 18\pi$

Cylinder volume $= 126\pi - 18\pi = 108\pi$

$\pi \times 3^2 \times h = 108\pi$
$h = 108 \div 9 = 12$
Answer: 12 cm