Day 145 **A Tier 10-a-day: Model Solutions**

1 Isosceles triangles and exterior angles

PQR is an isosceles triangle with PQ = PR.
QRS is a straight line.
Angle PRS = 129°.
Work out the size of angle x.
The diagram is not drawn accurately.

Angles on a straight line add up to 180°
Angle PRQ = 180 − 129 = 51°
Base angles are equal, so angle PQR = 51°
x = 180 − 51 − 51 = 78
Answer: x = 78°

2 HCF and LCM

(a) Write 84 and 132 as products of prime factors.
(b) Find the highest common factor (HCF) and the lowest common multiple (LCM) of 84 and 132.

(a) $84 = 2^2 \times 3 \times 7$ and $132 = 2^2 \times 3 \times 11$
(b) HCF: multiply the prime factors in both lists, taking the smaller power: $2^2 \times 3 = 12$
LCM: take the larger power of every prime: $2^2 \times 3 \times 7 \times 11 = 924$
Answer: (a) $84 = 2^2 \times 3 \times 7$, $132 = 2^2 \times 3 \times 11$ (b) HCF = 12, LCM = 924

3 Straight line graphs

A straight line has equation $-2x + 3y = 2$
(a) Find the gradient of the line.
(b) Find the coordinates of the point where the line crosses the x-axis.

(a) Rearrange into y = mx + c: $3y = 2x + 2$
$y = \frac{2}{3}x + \frac{2}{3}$
The gradient is $\frac{2}{3}$
(b) On the x-axis, y = 0: $-2x = 2$, so $x = -1$
Answer: (a) $\frac{2}{3}$ (b) $(-1, 0)$

4 Dividing fractions in context

A roll of ribbon is $5\frac{2}{5}$ metres long. Pieces of $\frac{3}{5}$ metre are cut from it.
How many pieces can be cut?

Divide: $5\frac{2}{5} \div \frac{3}{5}$
As improper fractions and multiplying by the reciprocal: $\frac{27}{5} \times \frac{5}{3}$
= 9
Answer: 9

5 Percentages in context

Leah earns £22000 a year.
She pays no tax on the first £12000.
She pays 30% tax on the rest.
Work out how much tax she pays in a year.

Taxable amount = £22000 − £12000 = £10000
Tax = 30% of £10000 = £3000
Answer: £3000

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6 Algebraic rearrangement and change of sign

$f(x) = x^3 - x^2 + 5x + 11$

Show that the equation $f(x) = 0$ has a solution between $x = -2$ and $x = -1$.

$f(-2) = -11$

$f(-1) = 4$

There is a change of sign and f is continuous, so there is a solution between $x = -2$ and $x = -1$

Answer: f(−2) = −11 and f(−1) = 4 (change of sign)

7 Mixed number addition and subtraction

Work out $3\frac{1}{3} - 1\frac{4}{7}$

Give your answer as a mixed number in its simplest form.

Write as improper fractions: $3\frac{1}{3} = \frac{10}{3}$ and $1\frac{4}{7} = \frac{11}{7}$

Common denominator 21: $\frac{70}{21} - \frac{33}{21}$

$= \frac{37}{21}$

$= 1\frac{16}{21}$

Answer: $1\frac{16}{21}$

8 Area of a trapezium

The area of this trapezium is 44 cm^2.

Work out the value of x.

The diagram is not drawn accurately.

$\frac{1}{2} \times (7 + 15) \times x = 44$

$11x = 44$

$x = 44 \div 11 = 4$

Answer: x = 4

9 Equations with algebraic fractions

Solve $\frac{6}{(x-5)} + \frac{6}{(x+4)} = 1$

Multiply every term by $(x - 5)(x + 4)$

$6(x + 4) + 6(x - 5) = (x - 5)(x + 4)$

$12x - 6 = x^2 - x - 20$

$x^2 - 13x - 14 = 0$

$(x + 1)(x - 14) = 0$

$x = -1$ or $x = 14$

Answer: x = −1 or x = 14

10 Capture-recapture

A scientist wants to estimate the number of frogs in a pond.

She catches 60 frogs, marks each one and returns them.

Later she catches a second sample of 40 frogs. 10 of these are marked.

Work out an estimate for the total number of frogs in a pond.

Proportion marked in the second sample = $\frac{10}{40}$

This is the same as the proportion in the whole population: $\frac{60}{N}$

$\frac{60}{N} = \frac{10}{40}$, so $N = 60 \times 40 \div 10 = 240$

Answer: 240