Day 144 **A Tier 10-a-day: Model Solutions**

1 Perimeter and area with algebra

An isosceles triangle has two equal sides of length $(4x - 2)$ cm and $(2x + 14)$ cm.
The third side is 9 cm.
Work out the perimeter of the triangle.

The two equal sides give an equation
$4x - 2 = 2x + 14$
$2x = 16$
$x = 8$
Equal sides $= 4 \times 8 - 2 = 30$ cm
Perimeter $= 30 + 30 + 9 = 69$
Answer: 69 cm

2 Exchange rates

The exchange rate is £1 = A$1.95. (Calculator)
A hotel room costs A$487.50 a night.
Work out the cost in pounds.

Divide by the exchange rate: $487.5 \div 1.95$
= £250
Answer: £250

3 Quadratic graph from its roots

The diagram shows the graph of $y = x^2 + bx + c$.
Find the value of b and the value of c.

The curve crosses the x-axis at $x = -1$ and $x = 1$
So $y = (x + 1)(x - 1)$
Expand: $y = x^2 - 1$
$b = 0$ and $c = -1$
Answer: b = 0, c = −1

4 HCF and LCM in index form

$A = 2^3 \times 3^4 \times 5^3$
$B = 2^3 \times 3 \times 5^2$
Find the highest common factor (HCF) and the lowest common multiple (LCM) of A and B.
Give your answers as products of prime factors.

HCF: for each prime take the lower power (a missing prime has power 0): $2^3 \times 3 \times 5^2$
LCM: for each prime take the higher power: $2^3 \times 3^4 \times 5^3$
Answer: HCF = $2^3 \times 3 \times 5^2$, LCM = $2^3 \times 3^4 \times 5^3$

5 Addition and multiplication rules

A spinner can land on red, blue or green.
P(red) = 0.1 and P(blue) = 0.5.
(a) Work out the probability that it lands on red or blue.
(b) Work out the probability that it lands on green.

(a) Mutually exclusive, so add: $0.1 + 0.5 = 0.6$
(b) $1 - 0.6 = 0.4$
Answer: (a) 0.6 (b) 0.4

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6 Rearranging formulae

Make w the subject of z = 9(w + 5)

Divide both sides by 9: $\frac{z}{9} = w + 5$

Subtract 5 from both sides

$w = \frac{z}{9} - 5$

Answer: $w = \frac{z}{9} - 5$

7 Exact trigonometric values

PQR is a right-angled triangle with the right angle at Q.
PQ = 10 cm and angle QPR = 30°.
Work out the exact length of QR.
Do not use a calculator.

QR is opposite and PQ is adjacent to the 30° angle, so use tan

$\tan 30° = \frac{\sqrt{3}}{3}$

$QR = 10 \times \frac{\sqrt{3}}{3} = \frac{10\sqrt{3}}{3}$

Answer: $\frac{10\sqrt{3}}{3}$ cm

8 Changing a ratio

A bag contains red counters and blue counters in the ratio 1 : 6.
8 more blue counters are added to the bag.
The ratio of red counters to blue counters is now 1 : 10.
How many counters were in the bag at the start?

Let red = 1x and blue = 6x. After adding: 1x : (6x + 8) = 1 : 10

$\frac{1x}{6x + 8} = \frac{1}{10}$, so 10 × 1x = 1(6x + 8)

10x = 6x + 8, so 4x = 8, x = 2

Start: 1x + 6x = 7 × 2 = 14 counters

Answer: 14

9 Square and cube numbers

n is the smallest positive integer such that 72 × n is a cube number.
Find the value of n.

Write 72 as a product of prime factors: $72 = 2^3 \times 3^2$

For a cube number every prime must have a power that is a multiple of 3.

Extra primes needed: 3 (needs 1 more)

n = 3

Answer: n = 3

10 Cumulative frequency graphs

The cumulative frequency graph shows information about the mass of 40 parcels.
Use the graph to find an estimate for
(a) the median,
(b) the number of parcels with a mass greater than 90 kg.

(a) Median at cumulative frequency 40 ÷ 2 = 20

Read across from 20 to the graph, then down to the axis

Median = 47.5 kg

(b) At 90 kg the cumulative frequency is 34

40 − 34 = 6

Answer: (a) 47.5 kg (b) 6