Day 143 **A Tier 10-a-day: Model Solutions**

1 Converting speeds

A sprinter travels at 4 m/s.
Work out this speed in kilometres per hour.

In 1 hour there are 3600 seconds, so 4 m/s = 4 × 3600 m/h
= 14400 m/h = 14400 ÷ 1000 km/h
= 14.4
Answer: 14.4 km/h

2 Simplifying algebraic fractions

Simplify fully

$$\frac{x^2 - 64}{x^2 + 8x}$$

Top: $x^2 - 64 = (x - 8)(x + 8)$
Bottom: $x^2 + 8x = x(x + 8)$
Cancel $(x + 8)$
Answer: $\mathbf{\frac{x - 8}{x}}$

3 Angles in parallel lines

The diagram shows two parallel lines crossed by a straight line.
(a) Work out the value of x.
(b) Work out the size of the angle marked $(3x + 15)°$.
The diagram is not drawn accurately.

(a) Corresponding angles are equal
$3x + 15 = 5x - 3$
$2x = 18$
$x = 9$
(b) $3 \times 9 + 15 = 42$
Answer: (a) x = 9 (b) 42°

4 LCM in context

Tom runs around a track in 50 seconds.
Isla runs around the same track in 70 seconds.
They start together at the start line and keep running.
How many laps will each have run when they are next together at the start line?

Find the LCM of 50 and 70: LCM = 350 seconds.
Tom: 350 ÷ 50 = 7 laps
Isla: 350 ÷ 70 = 5 laps
Answer: Tom 7 laps, Isla 5 laps

5 Trigonometry: finding an angle

Work out the size of angle x. (Calculator)
Give your answer to 1 decimal place.
The diagram is not drawn accurately.

The sides given are the opposite (13) and the adjacent (14), so use tan

$\tan x = \frac{13}{14}$

$x = \tan^{-1}(\frac{13}{14})$

$x = 42.9°$
Answer: x = 42.9°

Day 143 **A Tier 10-a-day: Model Solutions**

6 Quadratic graphs

The curve C has equation $y = x^2 + 4x - 12$
(a) Find the coordinates of the points where C crosses the x-axis.
(b) Find the coordinates of the turning point of C.

(a) Set $y = 0$: $x^2 + 4x - 12 = 0$
$(x + 6)(x - 2) = 0$
$x = -6$ or $x = 2$, so the graph crosses the x-axis at $(-6, 0)$ and $(2, 0)$
(b) The turning point is halfway between the roots: $x = (-6 + 2) \div 2 = -2$
$y = -16$, so the minimum point is $(-2, -16)$
Answer: (a) $(-6, 0)$ and $(2, 0)$ (b) $(-2, -16)$

7 Probability without replacement

A bag contains 6 mint and 5 fruit sweets.
Two sweets are taken at random without replacement.
Work out the probability that the two sweets are different colours.

P(same colour) $= \frac{6}{11} \times \frac{5}{10} + \frac{5}{11} \times \frac{4}{10} = \frac{5}{11}$

P(different) $= 1 - \frac{5}{11} = \frac{6}{11}$

Answer: $\frac{6}{11}$

8 Recurring decimals (mixed)

Write $0.35\dot{4}\dot{}$ as a fraction in its simplest form.

Let $x = 0.35454...$
Multiply by 10: $10x = 3.5454...$
Multiply by 1000: $1000x = 354.5454...$
Subtract: $990x = 354 - 3 = 351$
$x = \frac{351}{990} = \frac{39}{110}$

Answer: $\frac{39}{110}$

9 Area of shaded regions with circles

4 identical circles of radius 3 cm fit exactly inside a rectangle.
The circles touch each other and the sides of the rectangle. (Calculator)
Work out the area of the shaded region.
Give your answer to 3 significant figures.

Rectangle: $24 \times 6 = 144$ cm^2
4 circles: $4 \times \pi \times 3^2 = 36\pi$
Shaded area $= 144 - 36\pi = 30.9$
Answer: 30.9 cm^2

10 Rationalising with conjugates

Rationalise the denominator of $\frac{87}{6 - \sqrt{7}}$

Give your answer in the form $a + b\sqrt{c}$ where a and b are integers.

Multiply top and bottom by the conjugate $6 + \sqrt{7}$: $\frac{87(6 + \sqrt{7})}{(6 - \sqrt{7})(6 + \sqrt{7})}$

Denominator: $36 - 7 = 29$
$\frac{87(6 + \sqrt{7})}{29} = 3(6 + \sqrt{7})$
$= 18 + 3\sqrt{7}$
Answer: $18 + 3\sqrt{7}$