Day 137 **A Tier 10-a-day: Model Solutions**

1 Factorising

Factorise fully $4x^2 - 32x + 48$

Take out the common factor 4: $4x^2 - 32x + 48 = 4(x^2 - 8x + 12)$
Factorise the bracket: $4(x - 6)(x - 2)$
Answer: $4(x - 6)(x - 2)$

2 Exchange rates

The exchange rate is £1 = $1.25. (Calculator)
Femi changes £600 into US dollars.
How much does Femi get in US dollars?

£600 × 1.25 = $750
Answer: $750

3 Capture-recapture

A scientist wants to estimate the number of frogs in a pond.
She catches 60 frogs, marks each one and returns them.
Later she catches a second sample of 60 frogs. 9 of these are marked.
Work out an estimate for the total number of frogs in a pond.

Proportion marked in the second sample = $\frac{9}{60}$

This is the same as the proportion in the whole population: $\frac{60}{N}$

$\frac{60}{N} = \frac{9}{60}$, so N = 60 × 60 ÷ 9 = 400
Answer: 400

4 Pythagoras' theorem

Work out the length of x. (Calculator)
Give your answer to 3 significant figures.
The diagram is not drawn accurately.

$24^2 = x^2 + 14^2$
$x^2 = 576 - 196 = 380$
$x = \sqrt{380} = 19.5$
Answer: x = 19.5 cm

5 HCF and LCM in index form

$A = 2^3 \times 5^2 \times 7^4$
$B = 2 \times 5^2$
Find the highest common factor (HCF) and the lowest common multiple (LCM) of A and B.
Give your answers as products of prime factors.

HCF: for each prime take the lower power (a missing prime has power 0): 2×5^2
LCM: for each prime take the higher power: $2^3 \times 5^2 \times 7^4$
Answer: HCF = 2×5^2, LCM = $2^3 \times 5^2 \times 7^4$

6 Distance-time graphs

The graph shows the distance travelled by a cyclist.
(a) For how many minutes did the cyclist stop?
(b) Work out the speed during the final part of the journey. Give your answer in km/h.

(a) The graph is horizontal from 40 to 60 minutes, so the stop is 60 − 40 = 20 minutes
(b) Final part: distance 3 km in 15 minutes

Speed = distance ÷ time = 3 ÷ $\frac{15}{60}$ = 12 km/h

Answer: (a) 20 minutes (b) 12 km/h

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7 Surd fractions

Show that $\frac{\sqrt{175} - \sqrt{112}}{\sqrt{7}}$ is an integer, and find its value.

$\sqrt{175} = 5\sqrt{7}$ and $\sqrt{112} = 4\sqrt{7}$

Numerator $= 5\sqrt{7} - 4\sqrt{7} = \sqrt{7}$

$\frac{\sqrt{7}}{\sqrt{7}} = 1$

Answer: 1

8 Reading a histogram

The histogram shows information about the age of some items.

(a) Work out the number of items with a age in the class 0 to 10 years.

(b) Work out an estimate for the number with a age greater than 25 years.

(a) Frequency = frequency density × class width

$0.6 \times 10 = 6$

(b) 25 to 40: $0.8 \times 15 = 12$

Class 40 to 70: $0.4 \times 30 = 12$

Class 70 to 80: $2.2 \times 10 = 22$

Total $= 12 + 12 + 22 = 46$

Answer: (a) 6 (b) 46

9 Simultaneous equations (one quadratic)

Solve the simultaneous equations

$x^2 + y^2 = 169$

$y = x + 7$

Substitute $y = x + 7$ into $x^2 + y^2 = 169$

$x^2 + (x + 7)^2 = 169$

$2x^2 + 14x - 120 = 0$

Divide by 2: $x^2 + 7x - 60 = 0$

$(x + 12)(x - 5) = 0$

$x = -12$ or $x = 5$

Use $y = x + 7$ to find y: when $x = -12$, $y = -5$; when $x = 5$, $y = 12$

Answer: $x = -12$, $y = -5$ and $x = 5$, $y = 12$

10 Midpoints and length of a line segment

C is the point (3, 1) and D is the point (0, 5).

(a) Find the midpoint of CD.

(b) Work out the length of CD. Give your answer as a whole number.

(a) Midpoint $= \left(\frac{3 + 0}{2}, \frac{1 + 5}{2}\right) = (1.5, 3)$

(b) Length $= \sqrt{(-3)^2 + (4)^2}$

$= \sqrt{25} = 5$

Answer: (a) (1.5, 3) (b) 5