Day 136 **A Tier 10-a-day: Model Solutions**

1 Inequalities

Solve $-x - 3 > 3x + 13$

Collect the x terms on the left: $-4x - 3 > 13$
Add 3: $-4x > 16$
Divide by -4. Dividing by a negative number reverses the inequality sign
$x < -4$
Answer: $x < -4$

2 Circle theorems: angle at the centre

X, Y and Z are points on a circle with centre O.
Angle XZY = 67°.
Work out the size of angle x.
Give a reason for your answer.

The angle at the centre is twice the angle at the circumference
$x = 2 \times 67 = 134$
Answer: x = 134°

3 Reverse percentages

A watch costs £408 including VAT at 20%. (Calculator)
Work out the price of the watch before VAT is added.

Price including VAT is 120% of the price before VAT.
120% = £408, so 1% = £408 ÷ 120 = £3.40
100% = £3.40 × 100 = £340
Answer: £340

4 Pie charts

The pie chart shows how some students travel to school.
11 students chose car.
(a) Work out the angle for cycle.
(b) How many students chose cycle?

(a) $360 - 48 - 96 - 88 = 128°$
(b) 11 students is 88°, so 1 student is $88 \div 11 = 8°$
Cycle: $128 \div 8 = 16$
Answer: (a) 128° (b) 16

5 Column vectors

$\mathbf{a} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$.
Work out $4\mathbf{a} - 2\mathbf{b}$.

$4\mathbf{a} = \begin{pmatrix} 8 \\ -8 \end{pmatrix}$ and $2\mathbf{b} = \begin{pmatrix} 10 \\ -8 \end{pmatrix}$

$4\mathbf{a} - 2\mathbf{b} = \begin{pmatrix} 8 - 10 \\ -8 - (-8) \end{pmatrix}$

$= \begin{pmatrix} -2 \\ 0 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{-2} \\ \mathbf{0} \end{pmatrix}$

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6 Probability without replacement

A bag contains 7 red and 2 blue counters.
Two counters are taken at random without replacement.
Work out the probability that the two counters are the same colour.

P(red, red) = $\frac{7}{9} \times \frac{6}{8} = \frac{42}{72}$

P(blue, blue) = $\frac{2}{9} \times \frac{1}{8} = \frac{2}{72}$

Add: $\frac{42}{72} + \frac{2}{72} = \frac{11}{18}$

Answer: $\mathbf{\frac{11}{18}}$

7 Iteration

$x_{n+1} = \sqrt[3]{22 - 3x_n}$
$x_0 = 3$
Work out the values of x_1, x_2 and x_3.
Give your answers correct to 3 decimal places. (Calculator)

Substitute $x_0 = 3$ into the formula, then use each answer in the next step (keep the full calculator display)
$x_1 = 2.35133$
$x_2 = 2.46325$
$x_3 = 2.44466$
Answer: $x_1 = 2.351$, $x_2 = 2.463$, $x_3 = 2.445$

8 Square and cube numbers

n is the smallest positive integer such that 288 × n is a cube number.
Find the value of n.

Write 288 as a product of prime factors: $288 = 2^5 \times 3^2$
For a cube number every prime must have a power that is a multiple of 3.
Extra primes needed: 2 (needs 1 more), 3 (needs 1 more)
n = 6
Answer: n = 6

9 Bounds and accuracy

A cyclist travels 700 metres in 5.3 seconds. The distance is correct to the nearest 10 metres and the time to the nearest 0.1 second.
(a) Work out the upper bound and the lower bound of the speed in m/s.
(b) Write the speed to a suitable degree of accuracy. Give a reason for your answer.

(a) Upper bound = 705 ÷ 5.25 = 134.2857
Lower bound = 695 ÷ 5.35 = 129.9065
(b) Both bounds round to 130 to 2 s.f. but not to the same value to 3 s.f.
So the speed is 130 m/s to 2 significant figures.
Answer: (a) 134.2857, 129.9065 (b) 130 m/s (2 s.f.)

10 Probability with three selections

A bag contains 3 black pens and 6 blue pens.
Three pens are taken at random without replacement.
Work out the probability that all three pens are black.

P(all black) = $\frac{3}{9} \times \frac{2}{8} \times \frac{1}{7}$

= $\frac{6}{504} = \frac{1}{84}$

Answer: $\mathbf{\frac{1}{84}}$