Day 132 **A Tier 10-a-day: Model Solutions**

1 Pythagoras' theorem

Work out the length of x. (Calculator)
Give your answer to 3 significant figures.
The diagram is not drawn accurately.

$24^2 = x^2 + 22^2$

$x^2 = 576 - 484 = 92$

$x = \sqrt{92} = 9.59$

Answer: x = 9.59 cm

2 Percentage change

Ali buys a vintage radio for £40.
He sells it for £26.
Work out his percentage loss.

Loss = £40.00 − £26.00 = £14.00

Percentage loss = $\frac{£14.00}{£40.00} \times 100 = 35\%$

Answer: 35%

3 Converting units of area and volume

Convert 0.5 m^3 into cm^3.

1 m = 100 cm, so 1 m^3 = 100 × 100 × 100 = 1 000 000 cm^3
0.5 × 1 000 000 = 500000

Answer: 500000 cm^3

4 Counting outcomes

A PIN has 4 digits.
The first digit cannot be zero, but the other digits can be any of 0 to 9.
(a) How many different PINs are possible?
(b) Work out the probability of guessing a PIN in one attempt.

(a) 9 × 10 × 10 × 10 = 9000
(b) Only 1 of the 9000 PINs is correct

Answer: (a) 9000 (b) $\frac{1}{9000}$

5 Algebraic proof

Prove that the sum of three consecutive integers is always a multiple of 3.

Let the integers be n, n + 1 and n + 2
Sum = n + (n + 1) + (n + 2) = 3n + 3
= 3(n + 1), which is 3 × an integer, so it is a multiple of 3

Answer: Proved

6 Error intervals

x = 73 is rounded to 2 significant figures.
Write down the error interval for x.

The rounding unit is 1, so the value could be up to half of this, 0.5, either side.
Lower bound = 73 − 0.5 = 72.5, upper bound = 73 + 0.5 = 73.5

Answer: $72.5 \leq x < 73.5$

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7 Volume of cones and spheres

A sphere has radius 2 cm.
Work out the volume of the sphere. Give your answer in terms of π.

Volume = $\frac{4}{3} \times \pi \times r^3$

$= \frac{4}{3} \times \pi \times 2^3$

$= \frac{4}{3} \times 8\,\pi$

$= \frac{32}{3}\pi$

Answer: $\frac{32}{3}\pi$ cm^3

8 Successive percentage changes

The price of a games console is decreased by 40%.
The new price is then decreased by 25%.
Work out the overall percentage change.

First change: multiplier 0.6. Second change: multiplier 0.75
Overall multiplier = 0.6 × 0.75 = 0.45
0.45 − 1 = −0.55, so the overall change is 55%
Answer: 55% decrease

9 Rationalising with conjugates

Rationalise the denominator of $\frac{14}{4+\sqrt{2}}$

Give your answer in the form $a + b\sqrt{c}$ where a and b are integers.

Multiply top and bottom by the conjugate $4 - \sqrt{2}$: $\frac{14(4-\sqrt{2})}{(4+\sqrt{2})(4-\sqrt{2})}$

Denominator: 16 − 2 = 14

$\frac{14(4-\sqrt{2})}{14} = 1(4-\sqrt{2})$

$= 4 - \sqrt{2}$

Answer: $4 - \sqrt{2}$

10 Transformations of graphs

The graph of y = f(x) has a minimum point at (1, 3).
Write down the coordinates of the turning point of the graph of
(a) y = 2f(x)
(b) y = f(x + 3)

(a) Multiplying f(x) by 2 stretches the graph vertically by a factor of 2, so the y-coordinate is multiplied by 2
The minimum point is (1, 6)
(b) Replacing x by (x + 3) moves the graph 3 units left
The minimum point is (−2, 3)
Answer: (a) (1, 6) (b) (−2, 3)