Day 13 **A Tier 10-a-day: Model Solutions**

1 Exact trigonometric values

Without using a calculator, work out the exact value of
$3 \sin 45° + 5 \cos 30°$
Give your answer in its simplest form.

$\sin 45° = \frac{\sqrt{2}}{2}$ and $\cos 30° = \frac{\sqrt{3}}{2}$

$3 \times \frac{\sqrt{2}}{2} + 5 \times \frac{\sqrt{3}}{2} = \frac{3\sqrt{2} + 5\sqrt{3}}{2}$

Answer: $\mathbf{\frac{3\sqrt{2} + 5\sqrt{3}}{2}}$

2 Algebraic formulae in context

A mechanic charges £55 plus £40 for each hour of work.
(a) Write a formula for the total cost, £C, after t hours.
(b) Aisha pays £575 in total. Work out the number of hours.

(a) $C = 55 + 40t$
(b) $55 + 40t = 575$
$40t = 520$
$t = 13$
Answer: (a) C = 55 + 40t (b) 13

3 Circle theorems: isosceles triangles with radii

A, B and C are points on a circle with centre O.
Angle OAB = 52°.
(a) Work out angle AOB.
(b) Work out the size of angle x.
Give reasons for your answers.

(a) OA = OB (radii), so triangle OAB is isosceles and angle OBA = 52°
Angle AOB = 180 − 52 − 52 = 76°
(b) The angle at the centre is twice the angle at the circumference
x = 76 ÷ 2 = 38
Answer: (a) 76° (b) x = 38°

4 Rearranging formulae (powers and roots)

Make x the subject of $y = \frac{4}{x^2}$

Multiply both sides by x^2: $yx^2 = 4$

Divide by y: $x^2 = \frac{4}{y}$

Square root both sides (positive and negative roots)

Answer: $\mathbf{x = \pm\sqrt{\frac{4}{y}}}$

5 Vector geometry

OAB is a triangle. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.
M is the midpoint of AB.
Find, in terms of **a** and **b**,
(a) $\overrightarrow{AB}$ (b) $\overrightarrow{OM}$

(a) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\mathbf{a} + \mathbf{b}$

(b) $\overrightarrow{AM} = \frac{1}{2}\overrightarrow{AB} = \frac{1}{2}(\mathbf{b} - \mathbf{a})$

$\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM} = \mathbf{a} + \frac{1}{2}\mathbf{b} - \frac{1}{2}\mathbf{a} = \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

Answer: (a) $\mathbf{b - a}$ **(b)** $\mathbf{\frac{1}{2}a + \frac{1}{2}b}$

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6 Simplifying algebraic fractions

Simplify fully

$$\frac{x^2 - 9}{x^2 - 3x}$$

Top: $x^2 - 9 = (x - 3)(x + 3)$

Bottom: $x^2 - 3x = x(x - 3)$

Cancel $(x - 3)$

Answer: $\mathbf{\frac{x + 3}{x}}$

7 Successive percentage changes

The price of a games console is increased by 50%.
The new price is then decreased by 20%.
Work out the overall percentage change.

First change: multiplier 1.5. Second change: multiplier 0.8

Overall multiplier = $1.5 \times 0.8 = 1.2$

$1.2 - 1 = 0.2$, so the overall change is 20%

Answer: 20% increase

8 HCF and LCM

(a) Write 36 and 132 as products of prime factors.
(b) Find the highest common factor (HCF) and the lowest common multiple (LCM) of 36 and 132.

(a) $36 = 2^2 \times 3^2$ and $132 = 2^2 \times 3 \times 11$

(b) HCF: multiply the prime factors in both lists, taking the smaller power: $2^2 \times 3 = 12$

LCM: take the larger power of every prime: $2^2 \times 3^2 \times 11 = 396$

Answer: (a) $36 = 2^2 \times 3^2$, $132 = 2^2 \times 3 \times 11$ (b) HCF = 12, LCM = 396

9 Recurring decimals (mixed)

Write $0.9\dot{8}$ as a fraction in its simplest form.

Let $x = 0.988...$

Multiply by 10: $10x = 9.88...$

Multiply by 100: $100x = 98.88...$

Subtract: $90x = 98 - 9 = 89$

$x = \frac{89}{90}$

Answer: $\mathbf{\frac{89}{90}}$

10 Changing a ratio

A bag contains red counters and blue counters in the ratio 1 : 3.
18 more red counters are added to the bag.
The ratio of red counters to blue counters is now 11 : 6.
How many counters were in the bag at the start?

Let red = 1x and blue = 3x. After adding: $(1x + 18) : 3x = 11 : 6$

$\frac{1x + 18}{3x} = \frac{11}{6}$, so $6(1x + 18) = 11 \times 3x$

$6x + 108 = 33x$, so $27x = 108$, $x = 4$

Start: $1x + 3x = 4 \times 4 = 16$ counters

Answer: 16