Day 129 **A Tier 10-a-day: Model Solutions**

1 Primes

(a) Write down all the prime numbers between 40 and 60.
(b) Write 85 as the product of two prime numbers.

(a) A prime number has exactly two factors. Checking each number between 40 and 60 gives: 41, 43, 47, 53, 59
(b) 85 ÷ 5 = 17, and both 5 and 17 are prime.
Answer: (a) 41, 43, 47, 53, 59 (b) 5 × 17

2 Solving quadratic equations

Solve $4x^2 - 7x + 3 = 0$

Factorise: $(x - 1)(4x - 3) = 0$
$x - 1 = 0$ or $4x - 3 = 0$
$x = 1$ or $x = \frac{3}{4}$

Answer: $x = 1$ or $x = \frac{3}{4}$

3 Percentage change

The price of a games console changes from £75 to £131.25.
Work out the percentage increase.

Increase = £131.25 − £75.00 = £56.25
Percentage increase = $\frac{£56.25}{£75.00} \times 100 = 75\%$
Answer: 75%

4 Volume of a cylinder

A tin of soup is a cylinder with radius 3 cm and height 11 cm.
Work out the volume. Give your answer in terms of π.

Volume = $\pi r^2 h$
$= \pi \times 3^2 \times 11$
$= 99\pi$
Answer: 99π cm^3

5 Completing the square

Solve $x^2 - 8x + 6 = 0$ by completing the square.
Give your answers in surd form.

$x^2 - 8x + 6 = (x - 4)^2 - 10$
$(x - 4)^2 = 10$
$x - 4 = \pm\sqrt{10}$
$x = 4 \pm \sqrt{10}$
Answer: $x = 4 \pm \sqrt{10}$

6 Estimating the mean from grouped data

The table shows the distances, in km, travelled by 20 lorries.
(a) Write down the class that contains the median.
(b) Work out an estimate for the mean.

(a) The median is in position (20 + 1) ÷ 2 = 10.5. Cumulative frequencies: 5, 13, 16, 18, 20
The median is in the class $30 < d \leq 40$
(b) Midpoints: 25, 35, 45, 55, 65
25 × 5 + 35 × 8 + 45 × 3 + 55 × 2 + 65 × 2 = 780
Estimate = 780 ÷ 20 = 39
Answer: (a) $30 < d \leq 40$ (b) 39

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7 Recurring decimals, show that

(a) Show that $0.\dot{1}\dot{8} = \frac{2}{11}$

(b) Hence write $5.\dot{1}\dot{8}$ as a mixed number in its simplest form.

(a) Let x = 0.181818..., so 100x = 18.1818...

$100x - x = 18$, so $99x = 18$ and $x = \frac{18}{99} = \frac{2}{11}$

(b) $5.\dot{1}\dot{8} = 5 + \frac{2}{11} = 5\frac{2}{11}$

Answer: (b) $5\frac{2}{11}$

8 The cosine rule: finding a side

Work out the value of x. (Calculator)
Give your answer to 3 significant figures.
The diagram is not drawn accurately.

$x^2 = 16^2 + 19^2 - 2 \times 16 \times 19 \times \cos 122°$

$x^2 = 256 + 361 - 608 \cos 122°$

$x^2 = 939.19$

$x = 30.6$

Answer: x = 30.6 cm

9 Composite functions

f(x) = 2x − 4

$g(x) = x^2 + 4$

Solve gf(x) = 85

$gf(x) = (2x - 4)^2 + 4$

$(2x - 4)^2 + 4 = 85$, so $(2x - 4)^2 = 81$

$2x - 4 = \pm 9$

$2x - 4 = 9$ or $2x - 4 = -9$

$x = \frac{13}{2}$ or $x = -\frac{5}{2}$

Answer: $x = \frac{13}{2}$ or $x = -\frac{5}{2}$

10 Volume of composite solids

A solid is made from a cylinder and a hemisphere.
The cylinder has radius 6 cm and height 13 cm.
The hemisphere has the same radius as the cylinder and sits on top of it.
Work out the total volume. Give your answer in terms of π.

Cylinder = $\pi \times 6^2 \times 13 = 468\pi$

Hemisphere = $\frac{2}{3} \times \pi \times 6^3 = 144\pi$

Total = $468\pi + 144\pi = 612\pi$

Answer: 612π cm^3