Day 127 **A Tier 10-a-day: Model Solutions**

1 Equations with brackets

Solve $-5(x - 8) = 3(x - 2) - 2$

Expand the brackets: $-5x + 40 = 3x - 6 - 2$
Simplify: $-5x + 40 = 3x - 8$
Collect x terms on one side and numbers on the other: $-8x = -48$
$x = 6$
Answer: x = 6

2 Exchange rates

The exchange rate is £1 = €1.18. (Calculator)
A hotel room costs €259.60 a night.
Work out the cost in pounds.

Divide by the exchange rate: 259.6 ÷ 1.18
= £220
Answer: £220

3 Loci and constructions

PQ is a straight line of length 4 cm.
A point S moves so that it is always 2 cm from the line PQ.
Describe the locus of S.

Along the line, the locus is two parallel lines 2 cm away
Near each end, the locus is a semicircle of radius 2 cm centred on P and on Q
Answer: A 'racetrack' shape: two straight lines 2 cm either side of PQ joined by a semicircle of radius 2 cm at each end

4 Adding algebraic fractions

Write as a single fraction

$$\frac{-4}{(x-5)} + \frac{5}{(x-1)}$$

Common denominator is $(x - 5)(x - 1)$

$$= \frac{-4(x-1) + 5(x-5)}{(x-5)(x-1)}$$

$$= \frac{x - 21}{(x-5)(x-1)}$$

Answer: $\mathbf{\frac{x - 21}{(x-5)(x-1)}}$

5 Compound interest

Cara invests £500 in a savings account. (Calculator)
The account pays 2% compound interest per year.
Work out the amount in the account after 3 years.

Multiplier = 100% + 2% = 1.02
£500 × 1.02^3 = £530.604
= £530.60 to the nearest penny
Answer: £530.60

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6 Loci: area of a region

A horse is tied by a rope of length 8 m to the corner of a rectangular shed.
The shed measures 12 m by 11 m.
The horse can graze on any grass it can reach outside the shed.
Work out the area it can graze. Give your answer in terms of π.
The diagram is not drawn accurately.

The rope can sweep a sector of angle $360° - 90° = 270°$

Area $= \frac{270}{360} \times \pi \times 8^2$

$= \frac{3}{4} \times 64\pi$

$= 48\pi$

Answer: 48π m^2

7 Recurring decimals, show that

(a) Show that $0.\dot{4}\dot{1} = \frac{41}{99}$

(b) Hence write $1.\dot{4}\dot{1}$ as a mixed number in its simplest form.

(a) Let x = 0.414141..., so 100x = 41.4141...

$100x - x = 41$, so $99x = 41$ and $x = \frac{41}{99}$

(b) $1.\dot{4}\dot{1} = 1 + \frac{41}{99} = 1\frac{41}{99}$

Answer: (b) $1\frac{41}{99}$

8 Iteration

$x_{n+1} = \frac{6}{(x_n + 3)}$

$x_0 = 1$

Work out the values of x_1, x_2 and x_3.

Give your answers correct to 3 decimal places. (Calculator)

Substitute $x_0 = 1$ into the formula, then use each answer in the next step (keep the full calculator display)

$x_1 = 1.5$

$x_2 = 1.33333$

$x_3 = 1.38462$

Answer: $x_1 = 1.500$, $x_2 = 1.333$, $x_3 = 1.385$

9 Finding an interest rate

Owen invests £1000 in a savings account. (Calculator)
The account pays compound interest at a fixed rate each year.
After 2 years there is £1081.60 in the account.
Work out the annual rate of interest.

Let the yearly multiplier be m. Then $1000 \times m^2 = 1081.6$

$m^2 = 1081.6 \div 1000 = 1.0816$

$m = \sqrt{1.0816} = 1.04$

Rate $= (1.04 - 1) \times 100 = 4\%$

Answer: 4%

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10 Volume of composite solids

A solid is made from a cylinder and a cone.
The cylinder has radius 6 cm and height 8 cm.
The cone has the same radius as the cylinder and perpendicular height 8 cm.
Work out the total volume. Give your answer in terms of π.

Cylinder = $\pi \times 6^2 \times 8 = 288\pi$

Cone = $\frac{1}{3} \times \pi \times 6^2 \times 8 = 96\pi$

Total = $288\pi + 96\pi = 384\pi$

Answer: 384π cm^3