Day 126 **A Tier 10-a-day: Model Solutions**

1 Direct proportion

Here are the ingredients for 12 muffins.
Sugar: 120 g
Water: 900 ml
Flour: 360 g
(a) Work out how much sugar is needed for 3 muffins.
(b) Hannah has 831 ml of water. What is the greatest number of muffins she can make?

(a) sugar for one: 120 ÷ 12 = 10 g
For 3: 10 × 3 = 30 g
(b) water for one: 900 ÷ 12 = 75 ml
831 ÷ 75 = 11 remainder 6, so she can make 11
Answer: (a) 30 g (b) 11

2 Trigonometry: finding an angle

Work out the size of angle x. (Calculator)
Give your answer to 1 decimal place.
The diagram is not drawn accurately.

The sides given are the adjacent (6) and the hypotenuse (14), so use cos

$\cos x = \frac{6}{14}$

$x = \cos^{-1}(\frac{6}{14})$

$x = 64.6°$

Answer: x = 64.6°

3 Forming and solving equations

An isosceles triangle has two equal sides of length (3x − 6) cm and (x + 6) cm.
The third side is 9 cm.
Work out the perimeter of the triangle.

The two equal sides give 3x − 6 = x + 6
2x = 12
x = 6
Equal sides = 3 × 6 − 6 = 12 cm
Perimeter = 12 + 12 + 9 = 33
Answer: 33 cm

4 Converting units of area and volume

Convert 75000 cm^2 into m^2.

1 m^2 = 10 000 cm^2
75000 ÷ 10 000 = 7.5
Answer: 7.5 m^2

5 Quadratic graph from its roots

The diagram shows the graph of $y = -x^2 + bx + c$.
Find the value of b and the value of c.

The curve crosses the x-axis at x = −1 and x = 3
So y = −(x + 1)(x − 3)
Expand: $y = -x^2 + 2x + 3$
b = 2 and c = 3
Answer: b = 2, c = 3

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6 Recurring decimals to fractions

Write the recurring decimal $0.\dot{6}4\dot{3}$ as a fraction in its simplest form.

Let x = 0.643643643...
Multiply by 1000: 1000x = 643.643643...
Subtract: 999x = 643
$x = \frac{643}{999}$

Answer: $\mathbf{\frac{643}{999}}$

7 Algebraic rearrangement and change of sign

$f(x) = x^3 - 3x^2 + 7x - 4$
Show that the equation f(x) = 0 has a solution between x = 0 and x = 1.

f(0) = −4
f(1) = 1
There is a change of sign and f is continuous, so there is a solution between x = 0 and x = 1
Answer: f(0) = −4 and f(1) = 1 (change of sign)

8 Inverse proportion formulae

y is inversely proportional to x^2.
When x = 8, y = 6.
(a) Find a formula for y in terms of x.
(b) Work out the value of y when x = 2.

(a) $y = \frac{k}{x^2}$. Substitute: $6 = \frac{k}{64}$, so k = 6 × 64 = 384

$y = \frac{384}{x^2}$

(b) $y = \frac{384}{4} = 96$

Answer: (a) $\mathbf{y = \frac{384}{x^2}}$ **(b) 96**

9 The cosine rule: finding an angle

Work out the size of angle x. (Calculator)
Give your answer to 1 decimal place.
The diagram is not drawn accurately.

$\cos x = \frac{14^2 + 11^2 - 19^2}{2 \times 14 \times 11}$

$\cos x = \frac{-44}{308}$

$x = \cos^{-1}(-0.1429)$
x = 98.2°
Answer: x = 98.2°

10 Finding a rate of compound change

Zara invests £15,000 in a savings account. (Calculator)
Compound interest is paid at a fixed rate each year.
After 2 years the investment is worth £18,816.
Work out the annual rate of interest.

Overall multiplier = £18,816 ÷ £15,000 = 1.2544
Take the square root: $\sqrt{1.2544} = 1.12$
Rate = (1.12 − 1) × 100 = 12%
Answer: 12%